Preface

The three of us (Sara, Kim, and Linda) spend a lot of time with teachers, talking about how students are successful and what challenges they face. Over and over, we hear that students struggle with problem solving, especially with word problems. We see challenge expressed on teacher comments on Twitter, on Facebook, on community sites like MyNCTM, in conference sessions, even in the news. We hear this frustration from parents, from principals, and from math coaches across the country and internationally. We’ve written this book for classroom teachers and coaches who want to help their students have a more successful and meaningful approach to problem solving. If the approaches you have tried, such as using key words, or even using reading strategies to help students comprehend the problem, have only yielded spotty or unsatisfying results, this book is for you. If your students compute naked numbers efficiently, but when faced with a word problem they seem to pull numbers at random from problems and end up successfully calculating the wrong equations, this book is for you. If your students have ever looked at an equation and wondered how it fits the problem, this book is for you. We’ve written this book for all of the teachers whose students look at a word problem and say, “I just don’t get what they want me to do!”

How This Book Can Help You

Would you be surprised to know that every addition word problem you can think of can be classified into one of four categories? It’s true! The same is true for every subtraction problem. Multiplication and division are a bit more complicated, but not as much as you might think. The hard part about a word problem isn’t in using the operations (+ − × ÷) to compute an answer but, rather, it’s in figuring out which operation to use in a problem and why. Once you understand the four kinds of addition problem types and can recognize them in a problem’s story, the puzzle pieces can start to come together. We don’t mean to oversimplify the learning that needs to take place, because it isn’t simple, but we want you to know that there is something you can do to help students learn to tackle word problems productively. More important, we want students to tackle real problems that interest them and learn more mathematics as they do so.

Solving problems is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate, grapple with and solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking.

—Principles and Standards for School Mathematics
(National Council of Teachers of Mathematics, 2000, p. 52)

There is no magic elixir to solve the problem of word problems. This book shares our approach, which focuses on helping children mine the problem to uncover the underlying mathematics. Much needs to happen between the reading comprehension and computation stages of the problem-solving process. Yes, students need support to read and comprehend the words and language of the problem. Yes, they need to know how to compute the answer. But there is a whole
middle ground of exploration and understanding that students often rush through, where they could instead turn what they read into a solvable mathematical story and apply their operation sense to solve the puzzle. This is where we so often see a gap. We’ve written this book to fill that gap. We want students to see their world mathematically and know that mathematics can help them solve real problems. This is bigger (and more important) than passing a test.

To that end, this book is about problem solving. It’s about deciphering the kinds of word problems you see in normal, everyday lessons in classrooms like yours. It’s about the kinds of problems that are placed at the end of the lessons in your textbooks, the ones that your kids skip because they don’t know what to do. It’s about word problems. Story problems. Make-sense-of the-math, practice-a-skill problems. Sometimes these problems can seem artificial or contrived, but their straightforward simplicity is also a strength because they target the mathematical thinking we want students to develop. Wrestling productively with word problems can lay a foundation for the more complex and open-ended problems students will also encounter. These problems also have the potential to do and be more in their own right.

* Solutions to these (routine word) problems, particularly the solutions of younger children, do in fact involve real problem-solving behavior. … Word problems can provide insights into the development of more complex problem-solving abilities.  
  (Carpenter, 1985, p. 17)

### How to Use This Book

As a reader, you’ll get the most out of this book if you dig in and do the mathematics along the way. We’ve given you space to restate problems and draw pictures of your thinking with a focus on the mathematics. You’ll also find a collection of manipulatives helpful as you work through the book. Gather some counters, some base 10 blocks, and a fraction tool or two to use as aids while you solve problems. Think about how your students can use them too. Pictures and manipulatives can hold an idea in place right in front of you so that you can think about it more deeply. With manipulatives, you can do even more. You can make a change more quickly and easily: Manipulatives allow students to act out what happens in a problem, and to use attributes like color and size to highlight features of a problem. As you’ll see, the best tool for the job depends on the problem. There will be plenty of examples for you to explore.

There are places in the book where we’ve suggested you stop and talk with your colleagues. If you’re reading the book as part of a professional learning community, plan your discussions around these stopping points. You’ll find that there are plenty of opportunities for conversations about student thinking and the operations. But you’re never truly reading alone! Throughout each chapter you’ll find work samples inspired by student work. Several times in each chapter you’ll also find teacher commentary on the work samples. These comments are honest. Sometimes the teachers are bewildered, and sometimes the teachers are excited by what they see. Let these teacher voices be your companions as you tackle the new ideas. Use the teacher comments and the thinking breaks as reminders to take a pause and extend your own ideas a little further. The end of each chapter also has exercises and reflection questions that will help you and your colleagues connect what you’ve learned to your own classrooms.
After each chapter we also suggest that you look at the problems in your textbook and categorize them—not just for practice recognizing the structures you’ll soon learn about, but also to evaluate how much exposure your students are getting to the full range of problem types. If you discover that your textbook does not present enough variety, this book will give you the tools needed to make adjustments.

We recognize that many of the problems shared in this book will be unfamiliar contexts for your students. If you find yourself thinking that your students will not understand a problem context, we invite you to change the problem! Make sense of the problem situation yourself so that you know what mathematical features are important and then change the details. Even better, invite your students to craft and pose their own meaningful word problems to solve. With your new understanding of the problem situations, you will have all the tools you need to guide your students.

We have to be honest. The ideas in this book may challenge your current understandings of some mathematical ideas. At times, we will ask you to look at something you have been doing since you were eight years old and revisit it with new eyes. This may cause some disequilibrium, and it may be uncomfortable at first. It’s as if we are asking you to walk but by switching the foot you lead with. (Try it! It’s not easy!) When the familiar becomes unfamiliar, we encourage you to take a deep breath, trust us, and lead with the other foot. We’ll get you there. Here’s to lifelong learning!