Introduction

Dylan is a precocious fourth grader who loves mathematics. One of his favorite pastimes is playing the 24 Game (Suntex International Inc., 1988). For those of us not familiar with this particular game, Dylan will quickly show you that this competitive game involves a card containing four numbers (e.g., 7, 5, 4, and 3). Once the card is placed on the table, each player in the game tries to figure out how to make the number 24 using addition, subtraction, multiplication, and division. For the example with 7, 5, 4, and 3, Dylan gave the following answer:

\[
\begin{align*}
7 - 5 &= 2 \\
4 \times 3 &= 12 \\
12 \times 2 &= 24
\end{align*}
\]

In this specific example, Dylan rattled off the difference between seven and five, the product of four and three, and multiplied those two answers to get 24. To note, Dylan was able to solve this particular problem before the teacher had finished placing the card on the table.

Dylan demonstrates a high level of proficiency, or mastery, in procedural knowledge in the area of computation involving the four basic operations with single-digit whole numbers (e.g., additive thinking and multiplicative thinking). However, there is more to Dylan’s mathematics learning than his mastery of number facts. Dylan possesses a balance of conceptual understanding, procedural knowledge, and the ability to apply those concepts and thinking skills to a variety of mathematical contexts. By balance, we mean that no one dimension of mathematics learning is more important than the other two. Conceptual understanding, procedural knowledge, and the application of concepts
and thinking skills are each essential aspects of learning mathematics. Dylan’s prowess in the 24 Game is not the result of his teachers implementing procedural knowledge, conceptual understanding, and application in isolation, but through a series of linked learning experiences and challenging mathematical tasks that result in him engaging in both mathematical content and processes.

If you were to engage in a conversation with Dylan about mathematics, you would quickly see that he is able to discuss the concept of multiplication and describe different ways to represent multiplication (i.e., equal groups, arrays, and number line models). Furthermore, he can articulate which model he prefers and why: “I sometimes pick the model based on the type of problem. You know, some ways work better with certain problems.” Dylan also recognizes that he must apply this conceptual understanding and thinking to solving problems involving rates and price. He says, “If a pencil from the school store costs 10 cents and I want to buy five pencils, I need 50 cents.” Dylan also mentions that he could easily use this information when he learns about geometric measurements next year. “Well, that is what my teacher tells me,” he adds. Dylan’s mathematics learning is not by chance, but by design. His progression in conceptual understanding, procedural knowledge, and the application of concepts and thinking skills come from the purposeful, deliberate, and intentional decisions of his current and past teachers. These decisions focus on the following:

- What works best and what works best when in the teaching and learning of mathematics, and
- Building and supporting assessment-capable visible learners in mathematics.

This book explores the components in mathematics teaching and learning in Grades 3–5 through the lens of what works best in student learning at the surface, deep, and transfer phases. We fully acknowledge that not every student in your classroom is like Dylan. Our students come to our classrooms with different background knowledge, levels of readiness, and learning needs. Our goal is to unveil what works best so that your learners develop the tools needed for successful mathematics learning.
What Works Best

Identifying what works best draws from the key findings from Visible Learning (Hattie, 2009) and also guides the classrooms described in this book. One of those key findings is that there is no one way to teach mathematics or one best instructional strategy that works in all situations for all students, but there is compelling evidence for certain strategies and approaches that have a greater likelihood of helping students reach their learning goals. In this book, we use the effect size information that John Hattie has collected and analyzed over many years to inform how we transform the findings from the Visible Learning research into learning experiences and challenging mathematical tasks that are most likely to have the strongest influence on student learning.

For readers less familiar with Visible Learning, we would like to take a moment to review what we mean by what works best. The Visible Learning database is composed of over 1,800 meta-analyses of studies that include over 80,000 studies and 300 million students. Some have argued that it is the largest educational research database amassed to date. To make sense of so much data, John Hattie focused his work on meta-analyses. A meta-analysis is a statistical tool for combining findings from different studies, with the goal of identifying patterns that can inform practice. In other words, a meta-analysis is a study of studies. The mathematical tool that aggregates the information is an effect size and can be represented by Cohen’s $d$. An effect size is the magnitude, or relative size, of a given effect. Effect size information helps readers understand not only that something does or does not have an influence on learning but also the relative impact of that influence.

For example, imagine a hypothetical study in which pausing instruction to engage in a quick exercise or “brain break” results in relatively higher mathematics scores among fourth graders. Schools and classrooms around the country might feel compelled to devote significant time and energy to the development and implementation of brain breaks in all fourth grade classrooms in a specific district. However, let’s say the results of this hypothetical study also indicate that the use of brain breaks had an effect size of 0.02 in mathematics achievement over the control group, an effect size pretty close to zero. Furthermore, the large number of students participating in the study made it almost certain
there would be a difference in the two groups of students (those participating in brain breaks versus those not participating in brain breaks). As an administrator or teacher, would you still devote large amounts of professional learning and instructional time on brain breaks? How confident would you be in the impact or influence of your decision on mathematics achievement in your district or school?

This is where an effect size of 0.02 for the “brain breaks effect” is helpful in discerning what works best in mathematics teaching and learning. Understanding the effect size helps us know how powerful a given influence is in changing achievement—in other words, the impact for the effort or return on the investment. The effect size helps us understand not just what works, but what works best. With the increased frequency and intensity of mathematics initiatives, programs, and packaged curricula, deciphering where to best invest resources and time to achieve the greatest learning outcomes for all students is challenging and frustrating. For example, some programs or packaged curricula are hard to implement and have very little impact on student learning, whereas others are easy to implement but still have limited influence on student growth and achievement in mathematics. This is, of course, on top of a literacy program, science kits, and other demands on the time and energy of elementary school teachers. Teaching mathematics in the Visible Learning classroom involves searching for those things that have the greatest impact and produce the greatest gains in learning, some of which will be harder to implement and some of which will be easier to implement.

As we begin planning for our unit on rational numbers, knowing the effect size of different influences, strategies, actions, and approaches to teaching and learning proves helpful in deciding where to devote our planning time and resources. Is a particular approach (e.g., classroom discussion, exit tickets, use of calculators, a jigsaw activity, computer-assisted instruction, simulation creation, cooperative learning, instructional technology, presentation of clear success criteria, development of a rubric, etc.) worth the effort for the desired learning outcomes of that day, week, or unit? With the average effect size across all influences measuring 0.40, John Hattie was able to demonstrate that influences, strategies, actions, and approaches with an effect size greater than 0.40 allow students to learn at an appropriate rate, meaning at least a year of growth for a year in school. Effect sizes greater than 0.40 mean...
more than a year of growth for a year in school. Figure I.1 provides a visual representation of the range of effect sizes calculated in the Visible Learning research.

Before this level was established, teachers and researchers did not have a way to determine an acceptable threshold, and thus we continued to use weak practices, often supported by studies with statistically significant findings.

Consider the following examples. First, let us consider classroom discussion or the use of mathematical discourse (see NCTM, 1991). Should teachers devote resources and time into planning for the facilitation of classroom discussion? Will this approach to mathematics provide a return on investment rather than “chalk talk,” where we work out lots of problems on the board for students to include in their notes? With classroom discussion, teachers intentionally design and purposefully plan for learners to talk with their peers about specific problems or approaches to problems (e.g., comparing and contrasting strategies for multiplying and dividing large numbers versus small numbers,
explaining their development of a formula for a three-dimensional shape in collaborative groups. Peer groups might engage in working to solve complex problems or tasks (e.g., determining the equivalent decimal for a fraction using a number line). Although they are working in collaborative groups, the students would not be ability grouped. Instead, the teacher purposefully groups learners to ensure that there is academic diversity in each group as well as language support and varying degrees of interest and motivation. As can be seen in the barometer in Figure I.2, the effect size of classroom discussion is 0.82, which is well above our threshold and is likely to accelerate learning gains.

Therefore, individuals teaching mathematics in the Visible Learning classroom would use classroom discussions to understand mathematics learning through the eyes of their students and for students to see themselves as their own mathematic teachers.

Second, let us look at the use of calculators. Within academic circles, teacher workrooms, school hallways, and classrooms, there have been

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**Ability grouping**, also referred to as tracking or streaming, is the long-term grouping or tracking of learners based on their ability. This is different from flexibly grouping students to work on a specific concept, skill, or application or address a misconception.
many conversations about the use of the calculator in mathematics. There have been many efforts to reduce the reliance on calculators while at the same time developing technology-enhanced items on assessments in mathematics. Using a barometer as a visual representation of effect sizes, we see that the use of calculators has an overall effect size of 0.27. The barometer for the use of calculators is in Figure I.3.

As you can see, the effect size of 0.27 is below the zone of desired effects of 0.40. The evidence suggests that the impact of the use of calculators on mathematics achievement is low. However, closer examination of the five meta-analyses and the 222 studies that produced an overall effect size of 0.27 reveals a deeper story to the use of calculators. Calculators are most effective in the following circumstances: (1) when they are used for computation, deliberate practice, and learners checking their work; (2) when they are used to reduce the amount of cognitive load on learners as they engage in problem solving; and (3) when there is an intention behind using them (e.g., generating a pattern of square numbers, computing multiples of 10, or calculating the area or volume

of a large space or object). This leads us into a second key finding from John Hattie’s Visible Learning research: *We should not hold any influence, instructional strategy, action, or approach to teaching and learning in higher esteem than students’ learning.*

### What Works Best When

Visible Learning in the mathematics classroom is a continual evaluation of our impact on student learning. From the above example, the use of calculators is not really the issue and should not be our focus. Instead, our focus should be on the intended learning outcomes for that day and how calculators support that learning. Visible Learning is more than a checklist of dos and don’ts. Rather than checking influences with high effect sizes off the list and scratching out influences with low effect sizes, we should match the best strategy, action, or approach with learning needs of our students. In other words, is the use of calculators the right strategy or approach for the learners at the right time, for this specific content? Clarity about the learning intention brings into focus what the learning is for the day, why students are learning about this particular piece of content and process, and how we and our learners will know they have learned the content. Teaching mathematics in the Visible Learning classroom is not about a specific strategy, but a location in the learning process.

Visible Learning in the mathematics classroom occurs when teachers see learning through the eyes of their students and students see themselves as their own teachers. How do teachers of mathematics see multiplicative thinking, rational numbers, and geometric measurements through the eyes of their students? In turn, how do teachers develop assessment-capable visible learners—students who see themselves as their own teachers—in the study of numbers, operations, and relationships? Mathematics teaching and learning, where teachers see learning through the eyes of their learners and learners see themselves as their own teachers, results from specific, intentional, and purposeful decisions about each of these dimensions of mathematics instruction critical for student growth and achievement. Conceptualizing, implementing, and sustaining Visible Learning in the mathematics classroom by identifying what works best and what works best when is exactly what we set out to do in this book.
Over the next several chapters, we will show how to support mathematics learners in their pursuit of conceptual understanding, procedural knowledge, and application of concepts and thinking skills through the lens of what works best when. This requires us, as mathematics teachers, to be clear in our planning and preparation for each learning experience and challenging mathematics tasks. Using the guiding questions in Figure I.4, we will model how to blend what works best with what works best when. You can use these questions in your own planning. This planning guide is found also in Appendix B.

Through these specific, intentional, and purposeful decisions in our mathematics instruction, we pave the way for helping learners see themselves as their own teachers, thus making them assessment-capable visible learners in mathematics.

The Path to Assessment-Capable Visible Learners in Mathematics

Teaching mathematics in the Visible Learning classroom builds and supports assessment-capable visible learners (Frey, Hattie, & Fisher, 2018). With an effect size of 1.33, providing a mathematics learning environment that allows learners to see themselves as their own teacher is essential in today’s classrooms.

Ava is a bubbly fourth grader who loves school. She loves school for all of the right reasons—learning and socializing. At times, she confuses the two, but she quickly engages in the day’s mathematics lesson. During her review, Ava is engaging in the deliberate practice of adding fractions with unlike denominators. This is a topic that is challenging to her and is important background or prior knowledge for upcoming learning. During a discussion with her shoulder partner, Ava discusses her areas of strength and areas for growth: “I am good at adding fractions when the bottom numbers—wait, the denominators—are the same. You know, you just add the top numbers. I need more practice when the number—I mean, the denominator—is different. I have to slow down and figure it out.” This is a characteristic of an assessment-capable learner in mathematics.
I have to be clear about what content and practice or process standards I am using to plan for clarity. Am I using only mathematics standards or am I integrating other content standards (e.g., writing, reading, or science)?

Rather than what I want my students to be doing, this question focuses on the learning. What do the standards say my students should learn? The answer to this question generates the learning intentions for this particular content.

Once I have clear learning intentions, I must decide when and how to communicate them with my learners. Where does it best fit in the instructional block to introduce the day’s learning intentions? Am I going to use guiding questions?

As I gather evidence about my students’ learning progress, I need to establish what they should know, understand, and be able to do that would demonstrate to me that they have learned the content. This list of evidence generates the success criteria for the learning.

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**ESTABLISHING PURPOSE**

1. What are the key content standards I will focus on in this lesson?
   
   Content Standards:

2. What are the learning intentions (the goal and why of learning, stated in student-friendly language) I will focus on in this lesson?

   Content:
   Language:
   Social:

3. When will I introduce and reinforce the learning intention(s) so that students understand it, see the relevance, connect it to previous learning, and can clearly communicate it themselves?

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**SUCCESS CRITERIA**

4. What evidence shows that students have mastered the learning intention(s)? What criteria will I use?

   I can statements:

This planning guide is available for download at resources.corwin.com/vlmathematics-3-5.
Now I need to decide which tasks, activities, or strategies best support my learners. Will I use tasks that focus on conceptual understanding, procedural knowledge, and/or the application of concepts and thinking skills? What tools and problem-solving strategies will my learners have available?

I need to adjust the tasks so that all learners have access to the highest level of engagement. I can adjust the difficulty and/or complexity of a given task. What adjustments will I make to ensure all learners have access to the learning?

I need to create and/or gather the materials necessary for the learning experience (e.g., manipulatives, handouts, grouping cards, worked examples, etc.).

How will I check students’ understanding (assess learning) during instruction and make accommodations?

What activities and tasks will move students forward in their learning?

What resources (materials and sentence frames) are needed?

How will I organize and facilitate the learning? What questions will I ask? How will I initiate closure?

Once I have a clear learning intention and evidence of success, I must design my checks for understanding to monitor progress in learning (e.g., observations, exit tickets, student conferences, problem sets, questioning, etc.).

Finally, I need to decide how to manage the learning. How will I transition learners from one activity to the next? When will I use cooperative learning, small-group, or whole-group instruction? How will I group students for each activity?
Assessment-capable visible mathematics learners are:

1. Active in their mathematics learning. Learners deliberately and intentionally engage in learning mathematics content and processes by asking themselves questions, monitoring their own learning, and taking the reins of their learning. They know their current level of learning.

Later in the lesson, Ava is working in a cooperative learning group on finding the area of the school garden. Although the concept of area is a review, her teacher is using a concept Ava is familiar with to add context to two-by-two-digit multiplication. Her cooperative learning group has encountered a challenging calculation, $27 \times 16$. However, they quickly recognize that they have the tools to solve this problem. One of the group members chimes in, “To find the product, the answer to the problem, $27 \times 16$, I am going to use an open array model. These numbers are unfriendly.” This is a characteristic of an assessment-capable learner in mathematics.

Assessment-capable visible mathematics learners are:

2. Able to plan the immediate next steps in their mathematics learning within a given unit of study or topic. Because of the active role taken by an assessment-capable visible mathematics learner, these students can plan their next steps and select the right tools (e.g., manipulatives, problem-solving approaches, and/or metacognitive strategies) to guide their learning. They know what additional tools they need to successfully move forward in a task or topic.

Ava’s teacher, Ms. Christen Showker, takes time to individually conference with each student at least once a week. This allows the teacher to provide very specific feedback on each learner’s progress. Ava begins the conference by stating, “Yesterday’s exit ticket surprised me. You [Ms. Showker] wrote on my paper that I needed to revisit place value. I think I mixed up the thousands place. So, tomorrow I am going to work out the entire process for finding which number is larger in my notebook and not try and do it all in my head.” This is a characteristic of an assessment-capable learner in mathematics.
Assessment-capable visible mathematics learners are:

3. Aware of the purpose of the assessment and feedback provided by peers and the teacher. Whether the assessment is informal, formal, formative, or summative, assessment-capable visible mathematics learners have a firm understanding of the information behind each assessment and the feedback exchanged in the classroom. Put differently, these learners not only seek feedback, but they recognize that errors are opportunities for learning, monitor their progress, and adjust their learning (adapted from Frey et al., 2018) (see Figure I.5).

Over the next several chapters, we will explore how to create a classroom environment that focuses on learning and provides the best environment for developing assessment-capable visible mathematics learners who can engage in the mathematical habits of mind represented in one form or another in every standards document. Such learners can achieve the following:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning (© Copyright 2010. National Governors Association Center for Best Practices and Council of Chief State School Officers. All rights reserved.).

How This Book Works

As authors, we assume you have read Visible Learning for Mathematics (Hattie et al., 2017), so we are not going to recount all of the information
ASSESSMENT-CAPABLE VISIBLE LEARNERS

ASSESSMENT-CAPABLE LEARNERS:

**KNOW** THEIR CURRENT LEVEL OF UNDERSTANDING

**KNOW** WHERE THEY'RE GOING AND ARE CONFIDENT TO TAKE ON THE CHALLENGE

**SELECT** TOOLS TO GUIDE THEIR LEARNING

**SEEK** FEEDBACK AND RECOGNIZE THAT ERRORS ARE OPPORTUNITIES TO LEARN

**MONITOR** THEIR PROGRESS AND ADJUST THEIR LEARNING

**RECOGNIZE** THEIR LEARNING AND TEACH OTHERS

Source: Adapted from Frey, Hattie, & Fisher (2018).
Figure I.5
contained in that book. Rather, we are going to dive deeper into aspects of mathematics instruction in Grades 3–5 that are critical for students’ success, helping you to envision what a Visible Learning mathematics classroom like yours looks like. In each chapter, we profile three teachers who have worked to make mathematics learning visible for their students and have influenced learning in significant ways. Each chapter will do the following:

1. Provide effect sizes for specific influences, strategies, actions, and approaches to teaching and learning.
2. Provide support for specific strategies and approaches to teaching mathematics.
3. Incorporate content-specific examples from third, fourth, and fifth grade mathematics curricula.
4. Highlight aspects of assessment-capable visible learners.

Through the eyes of third, fourth, and fifth grade mathematics teachers, as well as the additional teachers and the instructional leaders in the accompanying videos, we aim to show you the mix and match of strategies you can use to orchestrate your lessons in order to help your students build their conceptual understanding, procedural knowledge, and application of concepts and thinking skills in the most visible ways possible—visible to you and to them. If you are a mathematics specialist, mathematics coordinator, or methods instructor, you may be interested in exploring the vertical progression of these content areas across preK–12 within Visible Learning classrooms and see how visible learners grow and progress across time and content areas. Although you may identify with one of the teachers from a content perspective, we encourage you to read all of the vignettes to get a full sense of the variety of choices you can make in your instruction, based on your instructional goals.

In Chapter 1, we focus on the aspects of mathematics instruction that must be included in each lesson. We explore the components of effective mathematics instruction (conceptual, procedural, and application) and note that there is a need to recognize that student learning has to occur at the surface, deep, and transfer levels within each of these
components. Surface, deep, and transfer learning served as the organizing feature of *Visible Learning for Mathematics*, and we will briefly review them and their value in learning. This book focuses on the ways in which teachers can develop students’ surface, deep, and transfer learning, specifically by supporting students, conceptual understanding, procedural knowledge, and application whether with comparing fractions or geometric measurement. Finally, Chapter 1 contains information about the use of checks for understanding to monitor student learning. Generating evidence of learning is important for both teachers and students in determining the impact of the learning experiences and challenging mathematical tasks on learning. If learning is not happening, then we must make adjustments.

Following this introductory chapter, we turn our attention, separately, to each component of mathematics teaching and learning. However, we will walk through the process starting with the application of concepts and thinking skills, then direct our attention to conceptual understanding, and finally, procedural knowledge. This seemingly unconventional approach will allow us to start by making the goal or endgame visible: learners applying mathematics concepts and thinking skills to other situations or contexts.

Chapter 2 focuses on *application* of concepts and thinking skills. Returning to our three profiled classrooms, we will look at how we plan, develop, and implement challenging mathematical tasks that scaffold student thinking as they apply their learning to new contexts or situations. Teaching mathematics in the Visible Learning classroom means supporting learners as they use mathematics in a variety of situations. In order for learners to effectively apply mathematical concepts and thinking skills to different situations, they must have strong conceptual understanding and procedural knowledge. Returning to Figure I.4, we will walk through the process for establishing clear learning intentions, defining evidence of learning, and developing challenging tasks that, as you have already come to expect, encourage learners to see themselves as their own teachers. Each chapter will discuss how to differentiate mathematical tasks by adjusting their difficulty and/or complexity, working to meet the needs of all learners in the mathematics classroom.

Chapters 3 and 4 take a similar approach with conceptual understanding and procedural knowledge, respectively. Using Chapter 2 as a reference
point, we will return to the three profiled classrooms and explore the conceptual understanding and procedural knowledge that provided the foundation for their learners applying ideas to different mathematical situations. For example, what influences, strategies, actions, and approaches support a learner's conceptual understanding of multiplication and division, rational numbers, or geometric measurement? With conceptual understanding, what works best as we encourage learners to see mathematics as more than a set of mnemonics and procedures? Supporting students' thinking as they focus on underlying conceptual principles and properties, rather than relying on memory cues like PEMDAS, also necessitates adjusting the difficulty and complexity of mathematics tasks. As in Chapter 2, we will talk about differentiating tasks by adjusting their difficulty and complexity.

In this book, we do not want to discourage the value of procedural knowledge. Although mathematics is more than procedural knowledge, developing skills in basic procedures is needed for later work in each area of mathematics from the area and circumference of a circle to linear equations. As in the previous two chapters, Chapter 4 will look at what works best when supporting students’ procedural knowledge. Adjusting the difficulty and complexity of tasks will once again help us meet the needs of all learners.

In the final chapter of this book, we focus on how to make mathematics learning visible through evaluation. Teachers must have clear knowledge of their impact so that they can adjust the learning environment. Learners must have clear knowledge about their own learning so that they can be active in the learning process, plan the next steps, and understand what is behind the assessment. What does evaluation look like so that teachers can use it to plan instruction and to determine the impact that they have on learning? As part of Chapter 5, we highlight the value of feedback and explore the ways in which teachers can provide effective feedback to students that is growth producing. Furthermore, we will highlight how learners can engage in self-regulation feedback and provide feedback to their peers.

This book contains information on critical aspects of mathematics instruction in Grades 3–5 that have evidence for their ability to influence student learning. We're not suggesting that these be implemented in isolation, but rather that they be combined into a series of linked
learning experiences that result in students engaging in mathematics learning more fully and deliberately than they did before. Whether finding equivalent fractions or calculating volume, we strive to create a mathematics classroom where we see learning through the eyes of our students and students see themselves as their own mathematics teachers. As learners progress from simplifying rational expressions to using ratios and proportions, teaching mathematics in the Visible Learning classroom should build and support assessment-capable visible mathematics learners.

Please allow us to introduce you to Christen Showker, Beth Buchholz, Hollins Mills, and Katy Campbell. These four elementary school teachers set out each day to deliberately, intentionally, and purposefully impact the mathematics learning of their students. Whether they teach third, fourth, or fifth grade, they recognize that:

- They have the capacity to select and implement various teaching and learning strategies that enhance their students’ learning in mathematics.
- The decisions they make about their teaching have an impact on students’ learning.
- Each student can learn mathematics, and they need to take responsibility to teach all learners.
- They must continuously question and monitor the impact of their teaching on student learning. (adapted from Hattie & Zierer, 2018)

Through the videos accompanying this book, you will meet additional elementary teachers and the instructional leaders who support them in their teaching. Collectively, the recognitions above—or their mindframes—lead to action in their mathematics classrooms and their actions lead to outcomes in student learning. This is where we begin our journey through Teaching Mathematics in the Visible Learning Classroom.