3. Planning Mathematics Lessons

The Four-Column Lesson Planning Model

A Quality Control Framework for Planning Lessons

Eliciting and Addressing Students’ Preconceptions

Building Conceptual and Procedural Knowledge

Supporting Metacognition

Alternatives to the Four-Column Lesson Plan Model

Ideas for Differentiating Instruction: Open Questions and Parallel Tasks

A “Top-10” List of Teaching Strategies And Tools

Reading Strategies

Writing Strategies

Manipulatives Usage

Calculator Usage

Cooperative Learning

Encouraging Student-Invented Strategies

Having Students Pose Problems

Lecture and Note Taking

Metaphor

Games
4. Mathematics Curriculum Models and Techniques

Recent Influential Curriculum Documents

- Political Context
- NCTM Curriculum Focal Points
  - Grade 6 Focal Points
  - Grade 7 Focal Points
  - Grade 8 Focal Points
- NCTM’s Focus in High School Mathematics
  - Number and Measurement
  - Algebraic Symbols
  - Functions
  - Geometry
  - Statistics and Probability
- Curricular Techniques in Mathematics
  - Use of Real-World Contexts
  - Teaching Through Problem Solving
  - Thematic Units
  - Integration of Content Strands
  - Guided Investigation
  - Progressive Formalization
- Technology Connection: Computer-Assisted Individualized Instruction
  - Drill and Practice
  - Teaching for Social Justice
  - Summary
- Choosing Curriculum Materials
  - Criteria for Examining Curricular Content
  - Examining Effects on Student Learning

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