less clear that the cross-price random effects \( u_{i2}, \ldots, u_{i4} \) are really needed. Estimation without these random effects reveals a DIC of 8148, so that the full random effects model is still slightly better (in terms of the DIC) than the reduced random effects model.

A somewhat unsatisfactory aspect of our models so far is that the type of nonlinearity of the price effects is determined in advance using a log-transformation of prices. To investigate whether the log-transformation is really appropriate we can replace the linear fixed effects in (4.12) by Bayesian P-splines (compare Section 4.2.2) leading to the model

\[
y_{ij} = \beta_0 + f_1(\text{log}p_{ij}) + f_2(\text{log}\text{pc}_{1ij}) + u_{i0} + u_{i1}\text{log}p_{ij} + u_{i2}\text{log}\text{pc}_{1ij} + u_{i3}\text{log}\text{pc}_{2ij} + u_{i4}\text{log}\text{pc}_{3ij} + \epsilon_{ij},
\]

(4.13)

We assumed a priori a monotonically decreasing effect of the own price and monotonically increasing effects of cross prices. Imposing monotonicity constraints is comparably straightforward in a Bayesian approach; see Brezger and Steiner (2008) for details. If the log-transformation is sufficient to capture the nonlinearity, the estimated curves \( f_1, \ldots, f_4 \) should be approximately linear. Figure 4.4 shows the estimated price effects. The plots reveal that the log-transformation is a reasonable approximation to the nonlinear effects, although considerable additional nonlinearity remains. The DIC of this model is 6606, which is approximately 1500 units below the strictly parametric models. This is a huge improvement in the goodness of fit and confirms our claim that there is considerable additional nonlinearity beyond the log-transformation.

The size of the random effects measured through the random effects variances is in the same range as in the parametric model (4.13). To demonstrate the outlet-specific heterogeneity, Figure 4.5 shows some outlet-specific log-sales own-price curves which are now additively composed of the nonlinear log-price effect \( f_1 \) and the linear log-price random effect \( u_{i1}\text{log}p_{ij} \).

A different approach to deal with nonlinearity of price effects and outlet-specific heterogeneity simultaneously has been proposed in Lang et al. (2011a). The paper assumes the model

\[
y_{ij} = \beta_0 + u_{i0} + f_1(p_{ij})(u_{i1} + 1) + f_2(\text{pc}_{1ij})(u_{i2} + 1) + f_3(\text{pc}_{2ij})(u_{i3} + 1) + f_4(\text{log}\text{pc}_{4ij})(u_{i4} + 1) + \epsilon_{ij},
\]