CHAPTER 3

Variability

LEARNING OBJECTIVES

After reading this chapter, you should be able to do the following:

• Explain what the standard deviation measures
• Compute the variance and the standard deviation for a population using a calculator
• Compute the variance and the standard deviation for a sample using a calculator and SPSS

POPULATION VARIABILITY

You have already learned that the mean is commonly used to summarize the center of a distribution of scores measured on an interval or ratio scale. While the mean does a good job describing the center of scores, it is also important to describe how “spread out from center” scores are. For example, imagine you are a psychologist studying individuals’ dispositional mood. Some people might have very little change in their mood from day to day, while others might have dramatically different moods on different days. In this case, the variability in someone’s mood can be really informative to researchers. To illustrate this point, consider two people who both complete the same “happiness scale” for 7 days in a row. A score of 0 = no happiness and a score of 15 = a lot of happiness. George’s daily happiness scores are 3, 3, 3, 3, 3, 3, and 3 for Monday through Sunday. In contrast, Morgan’s daily happiness scores are more variable. Her Monday through Sunday scores are 2, 2, 5, 2, 6, 4, and 0. Even though the centers of these two data sets are identical (i.e., \( \mu = 3 \) for both), you would certainly want to describe the different ways that George and Morgan experience mood. Thus, you need to describe the variability, or “spread,” of each person’s daily mood ratings. There are a number of ways to describe the variability of interval/ratio data. The easiest measure of variability is the range, which is the difference between the highest and lowest scores. For example, Morgan’s range is 6 – 0 = 6. The range is a poor measure of variability because it is very insensitive. By insensitive, we mean the range is unaffected by changes to any of the middle scores. As long as the highest score (i.e., 6) and the lowest score (i.e., 0) do not change, the range does not change. A sensitive measure of variability changes if any number in the distribution changes. Researchers value this sensitivity because it allows them to describe the variability in their data more precisely. The most common measure of variability is the standard deviation. The standard deviation tells you the typical, or standard, distance each score is from the mean. Therefore, the standard
deviation of George’s daily moods is 0 because all of the scores are exactly equal to the mean. In other words, George’s daily moods have zero variability. Morgan’s daily moods do vary, and therefore, the standard deviation of her data is larger (i.e., 1.93; you will learn to compute this next). Although the standard deviation is the preferred method of measuring variability, it can only be used when the data are interval/ratio. When the data are ordinal, you must use the range.

1. Why is the range a poor measure of variability?
   a. It uses only two values rather than all of the values in the distribution.
   b. It is overly sensitive to changes in the middle of the data.

2. What characteristic of a distribution of scores does a standard deviation describe?
   a. How far scores are from the mean
   b. How spread out the scores are
   c. The variability of scores in a distribution
   d. All of the above

3. The smallest standard deviation that is possible is ____ because this would mean that ____.
   a. –1; all of the scores are negative
   b. 0; all of the scores are the same
   c. 1; all of the scores are positive

4. What measure of variability should be used when the data are ordinal?
   a. Standard deviation
   b. Range

**STEPS IN COMPUTING A POPULATION’S STANDARD DEVIATION**

We are going to use Morgan’s moods to illustrate how to compute the standard deviation. Morgan’s daily moods are 2, 2, 5, 2, 6, 4, and 0. We are going to consider these seven scores to be a population because we are only interested in describing this one week of moods. Computing the standard deviation of this population consists of five steps. Focus on understanding what you are trying to do at each step rather than simply doing the calculations.

**Step 1: Compute the Deviation Scores** ($X - \mu$)

The standard deviation measures the standard (or typical) distance each score is from the mean. Thus, to compute the standard deviation, you first need to determine how far each score is from the mean. The distance each score is from the mean is called a deviation score and is computed as $X - \mu$, where $X$ is the score and $\mu$ is the mean of the population. For example, this small population of seven scores (2, 2, 5, 2, 6, 4, 0) has a mean of $\mu = 3$. Table 3.1 displays a deviation score for each of the seven scores in the population.
5. A deviation score measures 
   a. the typical distance all of the scores are from the mean.
   b. the distance of an individual score from the mean.

**Step 2: Square the Deviation Scores \( (X - \mu)^2 \)**

One logical way to find the typical deviation of scores from a mean is finding the average deviation score of a distribution. One could sum the deviation scores and divide their sum by the number of deviation scores, in this case 7. However, if you sum the deviation scores of any distribution, you get 0. Of course, if summing deviation scores always yields zero, this approach doesn’t help us differentiate between distributions with different amounts of variability. So we need some way to combine deviation scores without losing the variability among the scores. There are a number of ways to avoid this problem, but the one that statisticians use when computing the standard deviation is to square the deviation scores first and then to sum the squared deviation scores.\(^1\) The deviation scores have been squared in Table 3.2.

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**Table 3.1** Computing Deviation Scores, Step 1

<table>
<thead>
<tr>
<th>Score ((X))</th>
<th>Step 1: Deviation Score ((X - \mu))</th>
<th>(\text{Step 2: Squared Deviation Score } (X - \mu)^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>(2 - 3 = -1)</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>(2 - 3 = -1)</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>(5 - 3 = 2)</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>(2 - 3 = -1)</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>(6 - 3 = 3)</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>(4 - 3 = 1)</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>(0 - 3 = -3)</td>
<td>9</td>
</tr>
</tbody>
</table>

---

1 It is tempting to talk about the standard deviation as the average deviation from the mean, but this is not technically correct because the deviation scores always sum to zero and so the average deviation is 0. A different measure of variability is computed by taking the absolute value of the difference scores. This measure of variability is called the mean absolute deviation. However, the mean absolute deviation is rarely used. You will sometimes hear people talk about the standard deviation as the average deviation. Although this isn’t technically accurate, thinking about the standard deviation as the average deviation is fine.
reading question

6. Statisticians square each derivation score so that
   a. when they sum them they will not sum to zero.
   b. the standard deviation will be larger.

**Step 3: Compute the Sum of the Squared Deviation Scores, \(SS = \sum (X - \mu)^2\)**

Our goal is to compute the typical deviation score of the distribution of scores. Our next step is to compute the **sum of the squared deviation scores** (SS). To compute the SS, you simply add (i.e., sum) the squared deviation scores as was done in Table 3.3.

**Table 3.3 Computing SS With the Definitional Method, Step 3**

<table>
<thead>
<tr>
<th>Score (X)</th>
<th>Step 1: Deviation Score (X - (\mu))</th>
<th>Step 2: Squared Deviation Score (X - (\mu))^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2 - 3 = -1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2 - 3 = -1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>5 - 3 = 2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2 - 3 = -1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>6 - 3 = 3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>4 - 3 = 1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0 - 3 = -3</td>
<td>9</td>
</tr>
</tbody>
</table>

\[SS = \sum (X - \mu)^2 = 1 + 1 + 4 + 1 + 9 + 1 + 9 = 26\]

Table 3.3 illustrates how to compute the SS using what is called the **definitional formula**, \(\sum (X - \mu)^2\). There is another way to find the SS that, most of the time, is a lot easier. The second method uses the **computational formula**, \(\sum X^2 - \left(\frac{\sum X}{N}\right)^2\). Rather than individually computing every score’s deviation from the mean, squaring them all, and then summing them all, as the definitional formula requires, the computational formula allows you to find the SS with less arithmetic. The computational formula requires you to sum all of the original scores (i.e., the Xs) to find \(\sum X\), square every X, and then sum them all to find \(\sum X^2\). With this method, you don’t need to find each score’s deviation from the mean. The computations for this method are shown in Table 3.4.

The computational method and the definitional method will **ALWAYS** give you the same answer. However, we highly recommend the computational method. Once you get the hang of it, it is much faster. Furthermore, when the mean of the scores is not a whole number (e.g., 3.4578), the definitional formula not only is very tedious but also will lead to rounding error. So, you should work to become proficient with the computational method for finding the SS.
7. SS stands for the
   a. standard deviation.
   b. sum of the squared deviation scores.
   c. sum of the deviation scores.

8. The definitional method for finding the SS and the computational method for finding the SS will always provide the same value, but in most situations the ________ method is faster and will not reduce rounding error.
   a. Definitional method,
      \[ SS = \sum (X - \mu)^2 \]
   b. Computational method,
      \[ SS = \sum X^2 - \left( \frac{\sum X}{N} \right)^2 \]

Table 3.4  Computing SS With the Computational Method

<table>
<thead>
<tr>
<th>Score (X)</th>
<th>Square Scores X^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ \sum X = 21 \]
\[ \sum X^2 = 89 \]
\[ SS = \sum X^2 - \left( \frac{\sum X}{N} \right)^2 \]
\[ SS = \frac{89 - (21)^2}{7} \]
\[ SS = 89 - 63 = 26 \]

Step 4: Compute the Variance (σ²)

Again, our goal is to compute the typical, or standard, deviation of the scores from the mean in a distribution of scores. We cannot compute the average deviation score because their sum is always zero. So, instead, we compute the average squared deviation score, which is called the variance (σ², lowercase sigma squared). When computing any mean, we divide the sum of values by the number of values. Therefore, in this case, we divide the sum of the squared deviation scores by the number of squared deviations (i.e., N). The result is the mean of the squared deviation scores, the variance.

Population variance: \[ \sigma^2 = \frac{SS}{N} = \frac{26}{7} = 3.71. \]

9. The variance (σ²) is the
   a. typical squared deviation from the mean.
   b. typical deviation from the mean.

Step 5: Compute the Standard Deviation (σ)

We squared the deviation scores before we summed them and then divided the sum by N to get the variance. This means that the variance is the typical squared deviation of all the scores from the mean. While informative, the typical squared deviation from the mean is not very intuitive to think about. It is much easier to think about the typical deviation of scores from the mean. Therefore, we convert the
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typical squared deviation into the typical deviation by taking the square root of the variance. The square root of the variance is the typical or standard deviation of scores from the mean:

\[
\text{Population standard deviation: } \sigma = \sqrt{\sigma^2} = \sqrt{\frac{SS}{N}} = \sqrt{\frac{26}{7}} = \sqrt{5.71} = 1.93.
\]

The standard deviation tells us the standard (or typical) distance of all the scores from the mean. In this population, the typical distance of all the scores from the mean is 1.93. Some scores are more than 1.93 away from the mean and other scores are less than 1.93 away from the mean, but the “typical” distance of all the scores is 1.93.

10. The standard deviation (\(\sigma\)) is
   a. how far all of the scores are from the mean.
   b. the typical distance of all the scores from the mean; some scores will be further away and some closer, but this is the typical distance.

The five steps to computing the standard deviation of a population are listed in Table 3.4. It is worth familiarizing yourself with the verbal labels as well as their symbolic equivalents because we will be using both in future chapters. You should notice that there are two \(SS\) formulas. While these formulas are mathematically equivalent (meaning they yield the same answer), researchers use the second formula when working with larger data sets. This computational formula is much easier to use with large data sets than is the first definitional formula. You will use both of these equations in a future activity.

<table>
<thead>
<tr>
<th>Step</th>
<th>Verbal Label</th>
<th>Symbolic Equivalent</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Deviation score</td>
<td>((X - \mu))</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Square the deviation scores</td>
<td>((X - \mu)^2)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Sum of squared deviation scores</td>
<td>(SS)</td>
<td>Definitional: (SS = \sum (X - \mu)^2) &lt;br&gt;Computational: (SS = \sum X^2 - \left(\frac{\sum X}{N}\right)^2)</td>
</tr>
<tr>
<td>4</td>
<td>Population variance</td>
<td>(\sigma^2)</td>
<td>(\sigma^2 = \frac{SS}{N})</td>
</tr>
<tr>
<td>5</td>
<td>Population standard deviation</td>
<td>(\sigma)</td>
<td>(\sigma = \sqrt{\frac{SS}{N}})</td>
</tr>
</tbody>
</table>
11. What symbol represents the standard deviation of a population?
   a. SS  
   b. σ  
   c. σ²

12. Which equation defines the sum of the squared deviation scores?
   a. \( \sum (X - \mu)^2 \)  
   b. \( \sum (X - \mu) \)  
   c. \( \sqrt{\sigma^2} \)

13. Which equation is used for computing the \( SS \)?
   a. \( SS = \sum (X - \mu)^2 \)  
   b. \( SS = \sum X^2 - \frac{(\sum X)^2}{N} \)  
   c. \( \sigma = \sqrt{\frac{SS}{N}} \)

Once you have computed the standard deviation, you should interpret it in the context of the data set. For this population of Morgan’s daily moods, the happiness scores varied. In other words, Morgan was not equally happy every day. The standard deviation indicates how much her happiness varied across the week. Specifically, the standard deviation of 1.93 means that the typical distance of all the happiness scores from the mean of 3 was 1.93. With a mean of only 3, a standard deviation of 1.93 suggests that Morgan’s happiness scores varied quite a bit (e.g., 2, 2, 5, 2, 6, 4, 0).

It may help you understand that the standard deviation is actually measuring the typical distance of all the scores from the mean if we very briefly consider a completely new data set. Suppose Elliot’s average daily happiness score is 9. Specifically, his happiness scores on Monday through Sunday are 8, 8, 11, 8, 12, 10, and 6. Even though he is much happier than Morgan is on a daily basis, the standard deviation of Elliot’s daily moods is also 1.93. The standard deviations of these two data sets are identical because both data sets vary equally around their respective means of 3 and 9. Use the space in Table 3.5 to compute the standard deviation of the new data to confirm that it is 1.93.

<table>
<thead>
<tr>
<th>Score ( X )</th>
<th>Squared Score ( X^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>
Sum of squared deviations: \( SS = \sum x^2 - \frac{(\sum x)^2}{N} = \)

Population variance: \( \sigma^2 = \frac{SS}{N} = \)

Population standard deviation: \( \sigma = \sqrt{\frac{SS}{N}} = \)

14. In order for two data sets to have the same standard deviation, they must have the same mean.
   a. True
   b. False

**SAMPLE VARIABILITY**

Computing the variability of a sample of scores is very similar to computing the variability of a population of scores. In fact, there is only one computational difference that arises when you compute the variance (i.e., Step 4). To highlight the difference between the sample and population formulas, we will analyze the same scores we analyzed earlier (i.e., 2, 2, 5, 2, 6, 4, 0) as if they came from a sample rather than a population.

In the preceding example, we used Morgan’s daily moods on each day of a week as if it were a population because we were only trying to describe the variability of Morgan’s mood for that week. We were doing descriptive statistics because we were working with data from an entire population. If we wanted to describe the variability of Morgan’s moods during all of last year, but she did not complete the happiness scale for the entire year, we could use the week’s data we have as a sample to estimate the standard deviation of her moods for last year. In this scenario, the week of data we have are a sample from Morgan’s entire population of daily moods from last year. In this new scenario, we would be doing inferential statistics, and therefore, there is one small change to how we compute the standard deviation. The reason for the change is that we are using a sample to infer or estimate the value of the population’s standard deviation, and the change helps correct for sampling error.

15. When you are using a sample to estimate a population’s standard deviation, you are doing ________ statistics.
   a. descriptive
   b. inferential

16. When computing a sample’s standard deviation, there ________ to the computation process relative to when you are computing a population’s standard deviation.
   a. are many changes
   b. is one change
Steps 1 Through 3: Obtaining the SS

Computing the sum of the squared deviation scores (SS) is identical for a sample and population. The X scores were 2, 2, 5, 2, 6, 4, 0. Therefore, \( \sum X = 21 \) and \( \sum X^2 = 89 \).

\[
SS = \frac{\sum X^2 - (\sum X)^2}{N}
\]

\[
SS = \frac{89 - (21)^2}{7} = \frac{26}{6}
\]

\[
SS = 4.33.
\]

Step 4: Compute the Sample Variance \((SD^2)\)

Although the SS computations are the same, there is a difference between the computation of a sample variance and a population variance. To compute the population variance, you divided the SS by \( N \). To compute the sample variance, you divide the SS by \( N - 1 \). This is the only difference between the computation of a variance for a sample and a population:

Sample variance: \( SD^2 = \frac{SS}{N - 1} = \frac{26}{6} = 4.33 \).

Why we divide by \( N - 1 \) when using a sample to estimate a population’s variability rather than by \( N \) is a somewhat complicated issue. The simplest explanation is that samples are less variable than populations, and without the \( N - 1 \) adjustment, our variability estimate would be too low. For example, the variability of Morgan’s daily moods during a 7-day period, our sample, will be less than her moods during a 365-day period, our population. The small 7-day sample is going to have less variability than will the much larger 365-day population. More data tend to create more variability. The difference in variability between smaller samples and larger populations is a serious problem if you are trying to use a sample to estimate a population’s standard deviation. So, you need to do some kind of computational adjustment when using a sample to estimate a population’s variability; if you don’t, your variability estimate will tend to be too low. The computational adjustment statisticians determined to be most accurate in most situations is dividing the SS by \( N - 1 \) rather than by \( N \).

Reading Question

17. The SS is computed in exactly the same way for a sample and a population.
   a. True
   b. False

18. When using a sample to estimate a population’s variability, the SS is divided by \( N - 1 \) rather than by \( N \) to correct for a sample’s tendency to
   a. overestimate the variability of a population.
   b. underestimate the variability of a population.
Step 5: Compute the Sample Standard Deviation (SD)

Take the square root of the sample variance:

\[
\text{Sample standard deviation: } SD = \sqrt{\frac{SS}{N-1}} = \sqrt{\frac{26}{6}} = \sqrt{4.33} = 2.08.
\]

The verbal labels corresponding to each computational step for a sample’s standard deviation are identical to those used when computing a population’s standard deviation. However, as indicated above, the sample’s symbolic equivalents are Arabic letters rather than Greek letters (Table 3.6).

**Table 3.6** Summary of Five Steps to Computing a Sample’s Standard Deviation

<table>
<thead>
<tr>
<th>Step</th>
<th>Verbal Label</th>
<th>Symbolic Equivalent</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Deviation score</td>
<td>( (X - M) )</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Square the deviation scores</td>
<td>( (X - M)^2 )</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Sum of squared deviation scores</td>
<td>( SS )</td>
<td>( SS = \sum (X - M)^2 ) ( ) ( Computational: SS = \sum X^2 - \left( \sum X \right)^2 ) ( \frac{N}{N-1} )</td>
</tr>
<tr>
<td>4</td>
<td>Sample variance</td>
<td>( SD^2 )</td>
<td>( SD^2 = \frac{SS}{N-1} )</td>
</tr>
<tr>
<td>5</td>
<td>Sample standard deviation</td>
<td>( SD )</td>
<td>( SD = \sqrt{\frac{SS}{N-1}} )</td>
</tr>
</tbody>
</table>

19. What symbol represents the standard deviation of a sample?
   a. \( SD \)
   b. \( SD^2 \)
   c. \( SS \)

20. When *computing* the variance of an entire population, you are performing ___________ so divide the SS by ___.
   a. Descriptive statistics, \( N \)
   b. Inferential statistics, \( N - 1 \)

21. When *estimating* the variance of a population from a sample, you are performing ___________ so divide the SS by ___.
   a. Descriptive statistics, \( N \)
   b. Inferential statistics, \( N - 1 \)
You can compute the standard deviation and variance of a sample (not a population) using SPSS. Begin by entering the sample scores (2, 2, 5, 2, 6, 4, 0) into one column in SPSS. This is what your data file should look like when it is done (Figure 3.1):

![SPSS Screenshot of Data Entry Screen](image)

**Computing Measures of Variability**

- Click on the Analyze menu. Choose Descriptive Statistics and then Frequencies (see Figure 3.2).
  - You can obtain descriptive statistics (e.g., mean, standard deviation) in a lot of different ways in SPSS. We are only showing you one way here, but if you explore the menus, you can find other ways to obtain the same statistics.
- Move the variable(s) of interest into the Variable(s) box (see Figure 3.3).
- Make sure the Display Frequency Tables box is unchecked if you do not want a frequency distribution table.
- Click on the Statistics button.
- Click on the boxes for mean, standard deviation, variance, minimum, and maximum, and then click on the Continue button, and then click OK (see Figure 3.4).
- **Important Note:** SPSS computes the sample standard deviation and variance, not the population values.
Output

Your output file should look similar to the one below. Note that the results are the same as what you did by hand (Figure 3.5).

22. What is the standard deviation of these data?
   a. 3
   b. 2
   c. 2.08

23. SPSS can only be used to compute the standard deviation of a sample, not a population.
   a. True
   b. False
OVERVIEW OF THE ACTIVITY

In Activity 3.1, you will work with data sets to better understand variability and what causes variability in a data set. You will also practice computing the $SS$ and the standard deviation for populations and samples using your calculator.

Activity 3.1: Variability

Learning Objectives

After reading the chapter and completing this activity, you should be able to do the following:

- Recognize how measurement error, individual differences, and treatments can create variability
- Determine which of two distributions has a higher standard deviation by comparing histograms
- Explain what the standard deviation is to someone who has not taken a statistics course
- Compute the standard deviation for population or sample data presented in a histogram or a frequency table
- Use the definitional and computational formulas to compute the $SS$
- Use the statistics mode on your calculator to find the $\sum X^2$ and the $\sum X$
PART I: CONCEPTUAL UNDERSTANDING OF VARIABILITY

A developmental psychologist studying how the neonatal environment of infants affects their development is planning a huge national study. He needs to develop a reliable way to collect the physical measurements of newborn infants. He knows that the physical development of infants is carefully tracked during the child’s first year of life. At every doctor’s visit, the child’s height, weight, and head circumference are measured, typically by nurses. Consequently, he approaches nurses because he hopes that he can use the data they will collect from their future patients in his study. However, before he does, he wants to know how accurate their measurement procedures actually are. After getting the nurses’ agreement to participate, he brings a very realistic doll of a 1-year-old infant to the nurses and had each nurse measure the circumference of the infant’s head in centimeters. The 18 nurses’ measurements are graphed below.

1. Given that all nurses were measuring the head of the same doll, all of the head circumference measurements should be the same. In other words, there should be no variability in the measurements. However, the above graph clearly illustrates that there was variability in the measurements. Why was there variability in the measurements of the doll’s head circumference? (Select all that apply.)
   a. Some nurses held the tape measure tighter around the infants head while others held it looser.
   b. Each nurse put the tape measure in a slightly different place on the doll’s head
   c. Some nurses may have misread the tape measure.
   d. The doll’s head changed size between measurements.

2. In the above question, all of the variability in scores was created by measurement error because everyone was measuring the same thing and, therefore, should have obtained the same score. Unfortunately, measurement error is always present. No matter what you are measuring, you will never be able to measure it perfectly every time. You can, however, reduce the amount of measurement error. In the context of measuring an infant’s head circumference, how could the developmental psychologist and/or nurses reduce the variability in scores created by measurement error (i.e., what could they do to increase the accuracy/reliability of each measurement?). Select all that apply.
a. Give the nurses a lot of practice measuring different dolls’ heads.
b. Train the nurses to use a consistent degree of tension in the tape measure.
c. Use dolls with heads made out of a soft, pliable material.
d. Use a tape measure that only records centimeters, not inches.

The following week, different nurses measured the head circumference of 35 different infants. The head circumference measurements for 35 different infants are graphed below:

3. Did measurement error create some of the variability in scores that are graphed above?
   a. No, there is no measurement error. The head circumferences are only different because different infants were measured.
   b. Yes, there is measurement error. There is always some potential for measurement error any time a measurement is taken.

4. In addition to measurement error, something else is also creating variability in the distribution of 35 scores (i.e., head circumferences). Besides measurement error, what is another reason for the variability in the above distribution of 35 infants’ head circumferences? (Select all that apply)
   a. Different nurses measured the head circumferences and each nurse may have used a slightly different measurement technique.
   b. The 35 infants have heads that vary in size.

   In the previous question, the variability in head circumferences was created by both measurement error, which is always present, and the fact that the 35 infants’ heads actually varied in size. Researchers refer to this second source of variability as being created by individual differences. The fact that people are different from each other creates variability in their scores.

5. Using highly standardized measurement procedures can reduce the amount of variability created by ____________.
   a. individual differences
   b. measurement error
6. In which of the following distributions of scores would there be more variability created by individual differences?
   a. The heights of 50 first graders
   b. The heights of 50 elementary school children (first through fifth graders)

   It should be clear to you that researchers need to understand the variability in their data and what is creating it. Researchers view measurement error variability as “bad” and attempt to minimize it. Researchers also recognize that individual differences variability will always create variability in their data, and they try to control this variability with carefully designed experiments. In addition, in many research situations, researchers actually want to generate variability by creating different kinds of treatments.

   For example, suppose the researcher thought that physically touching prematurely born infants would increase their growth. To test this hypothesis, the researcher could conduct a study with two samples of prematurely born infants. All of the infants in Group 1 could be touched with skin-to-skin contact for at least 6 hours a day. All of the infants in Group 2 could be touched only by someone wearing gloves. After 4 weeks of these differing treatments, the circumferences of the babies’ heads could be compared.

7. In this study, there are three things creating variability in infants’ head circumference. The fact that measuring an infant’s head circumference is hard to do accurately contributes to the amount of ______ in this study.
   a. treatment differences variability
   b. individual differences variability
   c. measurement error variability

8. The fact that the researcher gave some infants 6 hours of skin-to-skin touch a day and some other infants no skin-to-skin touch contributes to the amount of ______ in this study.
   a. treatment differences variability
   b. individual differences variability
   c. measurement error variability

9. The fact that infants naturally differ from each other in head size contributes to the amount of ______ in this study.
   a. treatment differences variability
   b. individual differences variability
   c. measurement error variability

10. If we measured each of the following variables for every person in this class, which variables would have the most measurement error variability?
    a. Students’ report of their parents’ annual income
    b. Parents’ annual income recorded from official tax forms

11. If we measured each of the following variables for every person in this class, which variables would have the least individual differences variability?
    a. Number of siblings a person has
    b. Number of fingers a person has
12. Understanding variability is important because some variables simply have more variability than others do. For example, in high school students, which of the following variables would have the largest standard deviation?
   a. Annual income of parents
   b. Age

13. Which of the following variables would have the smallest standard deviation for high school students?
   a. Number of phone calls made in a day
   b. Number of phones owned

The following figure displays the head circumferences of 70 premature infants. Half of the infants were only touched by someone wearing gloves (the darker bars). The other half of the infants were only touched by someone who was not wearing gloves (the lighter bars).

14. In the above figure, the variability created by the different treatments (i.e., touching infants while wearing gloves vs. touching infants while not wearing gloves) is depicted by the fact that
   a. all of the infants who were touched while wearing gloves do not have the same head circumference.
   b. all of the infants who were touched while not wearing gloves do not have the same head circumference.
   c. the infants who were touched without wearing gloves (lighter bars) tended to have larger head circumferences than infants who were touched while wearing gloves (darker bars).
15. In most research situations, there will be variability that is created by measurement error, individual differences, and differing treatments. In the study described earlier, the researcher expected that touch would result in faster growth. Thus, the researcher compares the mean head circumference for a sample of the premature babies who were touched with direct skin contact to the mean head circumference for a sample of the premature babies who were only touched by someone wearing gloves. Suppose that the mean head circumference for the direct touch sample was 38 cm, and the mean for the other sample was 33 cm. Why can’t we just look at those two numbers and conclude that direct skin touching facilitated infant growth? Select all that apply.

a. The variability between the sample means may have been created by a treatment effect.

b. The variability between the sample means may have been created by individual differences.

c. The variability between the sample means may have been created by measurement error.

A primary goal of this course is to teach you how researchers determine if the variability you see in data (e.g., the difference between the head circumferences of infants who were touched in different ways) was likely created by a treatment difference or if the variability was likely created by individual differences and/or measurement error differences that exist between treatment conditions (i.e., sampling error).

16. The primary goal of this course is to teach you how to

a. design experiments to test treatment effects.

b. determine if variability is likely to be due to treatment effects or sampling error.

c. eliminate measurement error and individual difference variability.

As you know from the reading on variability, the standard deviation is commonly used to measure the amount of variability in a set of scores. Computing the standard deviation will not enable you to determine if the variability in the scores is created by treatment differences, individual differences, or measurement error. The standard deviation reveals the typical variability of scores from their mean. Later in the course, we will learn how to use other statistics to help us determine if treatment differences created variability in scores.

17. The standard deviation is a measure of

a. treatment variability in scores.

b. individual differences variability in scores.

c. typical distance of scores (i.e., variability) from the mean score.

If you understand the concept of variability, you should be able to “read” histograms. Specifically, you should be able to determine which of two histograms has more variability (i.e., a higher standard deviation).

For example, suppose that a professor asked students at the end of the semester how much they agree with the statement, “I enjoyed taking this course.” Students may respond with 1 = strongly agree, 2 = agree, 3 = neither agree nor disagree, 4 = disagree, or 5 = strongly disagree. Distributions from two of his classes are displayed on page 84. The first graph is from a research methods course, and the second graph is from a statistics course.
18. You should note that the mean rating for both courses was 3 (neither agree nor disagree). Which of the courses had more scores closer to its mean?
   a. Research methods
   b. Statistics

19. Given that the standard deviation measures the typical distance of scores from the mean, which course has the smaller standard deviation? (Hint: Both distributions have a mean of 3.)
   a. Research methods
   b. Statistics

There is far less variability in the research methods course than in the statistics course. In the research methods course, the majority of students responded with a 3, and most responses were very close to the mean. However, in the statistics course, most people responded with either a 1 or a 5, and most responses were relatively far from the mean. In other words, in the research methods course, most students gave the same answer (i.e., there was little variability in their responses). However, in the statistics course, there were greater differences of opinion (i.e., there was a lot of variability in responses). In general, graphs with a lot of data points “piled up” close to the mean (like the research methods distribution) have less variability (i.e., a smaller standard deviation) than graphs with a lot of data points “piled up” further from the mean (like the statistics distribution).

While there are other factors to consider, looking at where the scores pile up relative to the mean is a good way to start “reading” the variability in a distribution of scores. Use this rule to “read” the variability in the following pairs of graphs.

For Questions 20 to 22, determine if Graph A has more variability, Graph B has more variability, or if they have similar amounts of variability.
20.

Explain your choice:

![Graph A; Mean = 67.59](image)

![Graph B; Mean = 50](image)

21.

Explain your choice:

![Graph A; Mean = 33.75](image)

![Graph B; Mean = 86.25](image)
22. Explain your choice:

23. If a histogram has many scores piled up close to the mean value, the data set will tend to have
   a. a large standard deviation.
   b. a small standard deviation.

You should also note that there is no cutoff value for large or small standard deviations. In this case, the
standard deviation for the research methods course was $SD = 0.89$, and the standard deviation for the
statistics course was $SD = 1.78$. We can say that the standard deviation for the statistics class was relatively
large because a standard deviation of 1.78 is large when the range of possible responses is only between
1 and 5. A typical distance of 1.78 from the mean on a 5-point scale is quite large. However, if teaching
evaluations were made on a 50-point scale, a standard deviation of 1.78 would be quite small.

24. Which of the following standard deviations would represent greater variability relative to the
range of possible scores?
   a. A standard deviation of 2.51 for a variable measured with a 1-to-7 Likert scale
   b. A standard deviation of 2.51 for a fifth-grade spelling test (scores could potentially vary
      between 0 and 100)

**PART II: COMPUTING THE STANDARD DEVIATION**

A group of four students reports their heights in inches as follows: 68, 61, 72, 70.

25. Use the table below to help you compute the SS (sum of the squared deviation scores) using the
definitional formula:

\[ SS = \sum (X - M)^2 \]
### Chapter 3 Variability

#### 26. Although the definitional formula makes intuitive sense, it is not an easy formula to work with when you have a large set of scores. With large data sets, it is far easier to compute the SS using the computational formula:

$$SS = \frac{\sum X^2 - \left(\sum X\right)^2}{N}$$

The definitional and computational SS formulas will yield identical values. To avoid a common error when using the computational formula, be sure you understand the distinction between $\sum X^2$ and $(\sum X)^2$. To compute $\sum X^2$, you should square each score first, then sum them $(X^2 = 68^2 + 61^2 + 72^2 + 70^2)$. To compute $(\sum X)^2$, you should sum all the scores first and then square the sum $(\sum X)^2 = (68 + 61 + 72 + 70)^2)$. The $N$ is the number of scores. Use the table below to help you compute the SS using the computational formula.

<table>
<thead>
<tr>
<th>Score ($X$)</th>
<th>$X^2$</th>
<th>$SS = \frac{\sum X^2 - \left(\sum X\right)^2}{N}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>61</td>
<td></td>
<td></td>
</tr>
<tr>
<td>72</td>
<td></td>
<td></td>
</tr>
<tr>
<td>70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sum X = $</td>
<td>$\sum X^2 =$</td>
<td></td>
</tr>
</tbody>
</table>

This is a very small data set, so it is probably not obvious that the computational formula for the SS can save you quite a bit of time. When working with large data sets or when the mean of the data is not a whole number, the definitional formula takes longer, and the final answer is likely to have rounding error. Another advantage to using the computational formula is that even cheap statistics calculators will compute the $\sum X^2$ and $\sum X$ for you. Therefore, if you learn how to use your statistics calculator, computing the SS will become quite easy. You can simply substitute the values of $\sum X$ and $\sum X^2$ into the computational formula. Try to use the statistics mode on your calculator to find the $\sum X^2$ and $\sum X$. 

You should find that the $M$ is 67.75 and the $SS$ is 68.75.
27. Use the SS you computed in Question 26 to compute the standard deviation, assuming the data came from a sample. You will use the following equation whenever you are analyzing data from a sample. You should get 4.79.

\[ SD = \sqrt{\frac{SS}{N - 1}}. \]

28. Now use the SS you computed in Question 26 to compute the standard deviation, assuming that the data came from a population. You will use the following equation whenever you are analyzing data from an entire population. You should get 4.15.

\[ \sigma = \sqrt{\frac{SS}{N}}. \]

29. Figure out how to use the statistics mode on your calculator to compute the standard deviation of a population and a sample. There should be one button you can push or one line in a display that shows you the sample and population standard deviation. Don’t skip this. Finding the standard deviation with your calculator will be extremely helpful later in the course.

30. Compute the SS and the standard deviation for the following sample of five scores: 5, 6, 3, 2, 7.

<table>
<thead>
<tr>
<th>Score (X)</th>
<th>(X - M)</th>
<th>(X - M)^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ SS = \Sigma (X - M)^2 = \]
Confirm that you obtain the same results using the computational formula:

\[ SS = \sum X^2 - \frac{\left( \sum X \right)^2}{N}. \]

Compute the standard deviation:

\[ SD = \sqrt{\frac{SS}{N-1}}. \]

31. In the previous examples, you computed the SS using both the definitional and computational formulas. Although the definitional formula makes intuitive sense, it is far easier to use the computational formula. For all subsequent problems, you should use the computational formula. Compute the SS and the standard deviation for the following population of five scores: 1, 3, 3, 5, 7.

\[ SS = \sum X^2 - \frac{\left( \sum X \right)^2}{N}. \]

Compute the standard deviation:

\[ \sigma = \sqrt{\frac{SS}{N}}. \]

32. In most situations, which of the following formulas should you use to compute SS?

a. \[ SS = \sum X^2 - \frac{\left( \sum X \right)^2}{N} \]

b. \[ SS = \sum (X - M)^2 \]
33. The graph from the research methods course described earlier is reproduced below. Create a frequency distribution table from these population data.

<table>
<thead>
<tr>
<th>Research Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

34. Compute the standard deviation of the ratings in the research methods course.

35. You should have found that the standard deviation was 0.87. What does 0.87 mean in the context of this set of data?
   a. The typical distance between scores is .87.
   b. The typical distance between the scores and the mean is .87.
   c. The typical distance between the sample means is .87.

36. After computing the standard deviations for the research methods course, the instructor realizes that some students did not complete the rating form, and so it was a sample, not an entire population. Recompute the standard deviation of the course as if it came from a sample.
Chapter 3 Practice Test

1. A teacher asks nine of his students to read a book for 15 minutes, and he records the number of lines of text each student reads during that 15 minutes. The results are as follows:
   20, 13, 33, 11, 40, 29, 15, 38, 21
   What is the mean for this sample of nine students?
   a. 22.00
   b. 24.44
   c. 10.91
   d. 22.68

2. What is the deviation score for the person who read 20 pages?
   a. 24.44
   b. –24.44
   c. –4.44
   d. 4.44

3. What is the standard deviation for this sample of nine students?
   a. 24.44
   b. 10.91
   c. 10.29
   d. 220
   e. 952.22
   f. 14.88

4. Which of the following statements is the best interpretation of a standard deviation?
   a. The typical distance deviation scores are from the mean
   b. The typical distance scores are from each other
   c. The typical distance scores are from the deviation scores
   d. The typical distance scores are from the mean

5. What is SS?
   a. The sum of the squared deviation scores
   b. The sum of the squared scores
   c. The sum of the squared standard deviations
   d. The square of the summed scores

6. When is the standard deviation not an appropriate measure of variability?
   a. When the data are nominal or ordinal
   b. When the data are interval or ratio
   c. When the data are leptokurtic
   d. When the data are normally distributed
7. Compute the standard deviation for this population of scores:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>2</td>
</tr>
<tr>
<td>90</td>
<td>5</td>
</tr>
<tr>
<td>80</td>
<td>3</td>
</tr>
<tr>
<td>70</td>
<td>0</td>
</tr>
<tr>
<td>60</td>
<td>1</td>
</tr>
</tbody>
</table>

- a. 11.20
- b. 86.36
- c. 950
- d. 10.68

8. An instructor gives a multiple-choice exam that is graded electronically. What type of error is this instructor minimizing by using a multiple-choice, electronically graded exam?
- a. Individual differences
- b. Treatment effects
- c. Measurement error
- d. All of the above sources of variability would be reduced.

9. Which of the following variables would have more variability due to individual differences?
- a. SAT scores of college students in the nation
- b. SAT scores of college students at your college

10. The scores on the first exam in a statistics course had a mean of 78.32 with a standard deviation of 13.24. Scores on the second exam had a mean of 80 with a standard deviation of 11.32. For which exam were scores closer to the mean?
- a. Exam 1
- b. Exam 2

11. Scores on an exam for students in the same section of a chemistry course had a mean of 91.34 with a standard deviation of 14.63. Which of the following are sources of the variability in the exam scores? You may select more than one.
- a. Individual differences
- b. Treatment effects
- c. Measurement error

12. At the beginning of the school year, all of the students in three third-grade classes take a test to assess their current math skills. The mean and the standard deviation for each class are as follows:

Class 1: Mean = 73.59; SD = 24.78
Class 2: Mean = 65.42; SD = 8.43
Class 3: Mean = 85.32; SD = 22.86
Which class do you think will be easiest to teach?

a. Class 1. The standard deviation is highest in Class 1, indicating that the students in that Class have higher math skills than the students in the other classes.

b. Class 2. The standard deviation is lower for this class than Class 1 or 3. This suggests that the students are more homogeneous (similar) in their math skills and that it will be easier for the teacher to create lessons that will work for most of the students.

c. Class 3. The mean test score was highest for this group of students, which suggests that these students have the highest math skills and the teacher can create lessons that will work well for this group of high-ability students.

13. Can you use the statistics mode on your calculator to compute the mean and standard deviation for a set of data?

a. Yes, no problem!

b. I can do it if I have a set of instructions in front of me.

c. No, I haven’t been able to figure it out yet.