

# 1 Mindset, mastery and mistakes

Understanding how children learn mathematics is a critical element of your beginning to understand the errors and misconceptions they make and how you can embrace these in your teaching to support every child's mathematical cognitive development.

## Mathematical mistakes

A teacher's response to dealing with a child's mathematical error demands skill in diagnostic terms: different responses will be appropriate depending upon the nature (and frequency) of the error observed. Before reading on, make a list of the reasons why you think children might make mistakes and then compare them to the list below.

A mistake could be made for many reasons. It could be the result of carelessness, misinterpretation of symbols or text, inability to comprehend what the task is asking, misunderstanding the initial instructions, error in transforming a word problem to a mathematical problem, misunderstanding a keyword, an error in selecting the correct information to use, taking into account the problem's context without regard to the mathematics, using incorrect operations, lack of relevant experience or knowledge related to that mathematical topic/learning objective/concept, error in providing the solution in the correct context, leaving a task unfinished, a lack of awareness or inability to check the solution given, or the result of a misconception (adapted from Wijaya et al., 2014).

## Misconceptions and errors

The term 'misconception' is commonly used when a learner's conception is considered to be in conflict with the accepted meanings and understandings in mathematics (Barmby et al., 2009a). A misconception could be the misapplication of a rule, an over- or under-generalisation or an alternative conception of the situation. For example, *a number with three digits is bigger than a number with two digits* works in some situations (e.g. 328 is bigger than 35) but not necessarily in others where decimals are involved (e.g. 3.28 is not bigger than 3.5).

On the other hand, an error is principally formed within surface levels of knowledge: as such, a child's response to a task is procedural and can be corrected by the teacher providing correct alternatives (Ryan and Williams, 2007).

## Why do children have misconceptions?

Constructivist and constructionist theories explain why we all make mistakes and why they are a natural component of learning. Although the terminology used in each theory differs, they all commonly refer to children making systematic associations that they develop from their experiences within the rich world around them. The systematic associations are often thought of as a web, where connections are made between the particular instances of a concept that children experience. As these instances become increasingly associated, prototypes (early concepts) are formed (diSessa and Sherin, 1998; Tennyson, 1996). Noss and Hoyles (1996) see the web more broadly, to include the construction of a web of connections between resources which may be formal or informal, internal (cognitive) and external (physical or virtual). These views of children's early conceptual development rely on children noticing what is happening around them and (usually subconsciously) making connections with other experiences or understanding of the ideas being developed.

Up until recently, we could only observe external products of the process of conceptual change. However, with recent developments in technology, neuroscientists are able to observe connections in the brain being created in real time. The brain contains around 100 billion nerve cells called neurons. Neurons send messages to other neurons between gaps (synapses) into other neurons and it is this synaptic activity that creates long-lasting connections in the brain to create the webs discussed above. If the messages are only passed between synapses once or twice, the connections will disappear over time, but long-lasting connections remain with regular use, and connections will be created within different parts of the brain (Morgan, 2013).

As children develop their own understanding of the world they will unavoidably misconceive some ideas. When the connections are weak or poor, it is more common for mistakes due to misconceptions to occur. For example, if a child's experience of multiplying by ten has been to 'add a zero' then when multiplying 2.9 by 10 it is clear why the solution given is 2.90. When connections are stronger, children are able to think more efficiently and confidently as the conceptions become *compressed* (Dubinsky et al., 2005; Gray and Tall, 2007) or *reified* (Sfard, 1991) to become objects that can be more easily accessed. Interestingly, we know that despite what they are taught, children seem to make the same mathematical errors and construct their own alternative meanings for mathematics all over the world (Swan, 2001). It is important to note that misconceptions are not limited to children who need additional support: 'more able' children also make incorrect generalisations. Adults also possess misconceptions, and you will find many examples of world-renowned mathematicians making errors due to misconceptions by undertaking a simple Internet search.

Mistakes caused by misconceptions in mathematics are far more problematic than errors because they are set within deeper levels of knowledge (Ryan and Williams, 2007, p16). They demand diagnosis and dialogue to ascertain the misconception and this can be time-consuming for teachers. Nevertheless, Barmby et al. (2009b, p4)

argue that misconceptions must be regarded as *evolving understandings in mathematics . . . essential and productive for the development of more sophisticated conceptions and understanding.*

## Should we teach to avoid children making mistakes?

Teachers want their pupils to feel successful in mathematics. It is not uncommon for teachers who lack confidence in their own mathematical abilities to feel that the way to do this is to ensure that their pupils receive tasks where they do not make mistakes. However, this is the worst possible thing they could be doing!

There is no doubt that some mathematical errors could be avoided by teacher awareness, skilful choice of task and clarity of explanation: we will discuss these later in this chapter. However, previous editions of this book have made a concerted effort to see mistakes not as a failure in learning or teaching, but instead as part of the *learning which is more effective when common misconceptions are addressed, exposed and discussed in teaching* (Askew and Wiliam, 1995) by providing *focused teaching activities which tackle fundamental errors and misconceptions that are preventing progress* (DfES, 2005a, page 6). As Dowker (2009) points out, the focus here is one of using teacher assessment to *correct* identified errors and misconceptions. In order to use Assessment for Learning (AfL) effectively, emphasis also needs to be placed on children recognising themselves that errors and misconceptions are part of the learning process. Lee (2006) argues that such strategies lead to more creative approaches to AfL, help pupils see themselves as successful learners, and encourage both teachers and pupils to attempt more challenging work. Rather than simply correcting an error or misconception, it would appear far more productive for teachers to investigate the reason a child provides a given answer. Indeed, Barmby et al. (2009b, p5) maintain that *identifying and building on incomplete or incorrect conceptions are important ways of developing coherent mathematical knowledge* and Brodie (2014, p226) explains that errors are reasoned and reasonable for children, and argues that focusing on errors *allows teachers to develop nuanced understandings of the nature of mathematics, of teaching and learning and the relationships between them.*

Opinions on whether this is possible or even desirable differed according to the varying perspectives of primary children and their teachers. Some report that when primary children were asked how they felt about making mathematical mistakes, they expressed strong feelings of anger, frustration and disappointment (Bray, 2011; Koshy, 2000). In contrast, Cockburn (1999) and Koshy (2000) both reflected a growing view in the research evidence that mathematical errors can provide a useful insight for teachers into a child's thinking and understanding, an effective mechanism for assessment for learning and, with sensitive handling, can enable children to learn from mathematical mistakes (Cockburn, 1999; Heinze and Reiss, 2007; Steuer et al., 2013). It is from this standpoint that we present the remainder of this chapter.

## Why is making mistakes a good thing?

*Misconceptions are a natural outcome of intelligent mathematical development involving connections, generalisations, and concept formation . . . they signal a learning opportunity or zone and so, potential for development – for example – through targeted teaching.*

(Ryan and Williams, 2007, p270)

Swan (2001) suggests that far from trying to teach to avoid children developing misconceptions, the latter should be viewed as helpful and, possibly, 'necessary' stages in children's mathematical development. This suggests that a focus on how children are taught mathematics, rather than on what mathematics they are taught, is needed.

Revolutionary research methods in neuroscience allow us to see what happens in the brain's mechanisms when mistakes are made. This method uses event-related brain potentials (ERPs) that measure cognitive processes (Luck, 2004). We now have scientific evidence that whenever we make a mistake (be it knowingly or unknowingly), there is increased brain electrical activity because the brain experiences conflict between a correct solution and a mistake, compared to getting the correct answers. Furthermore, when an error is highlighted, a further ERP response (a Pe) occurs over a number of brain regions (Moser et al., 2011). We cannot overlook this finding: *the brain grows when we make a mistake, even if we are not aware of it, because it is a time of struggle; the brain is challenged, and this is the time the brain grows the most* (Boaler, 2016, p11). It is essential we embrace mistakes in learning and teaching: we return to this idea later in this chapter.

## Mathematical mindsets

For many years the general public and many teachers thought that people are born with a fixed intelligence. We accepted talk about talented sportspeople, genius minds and gifted musicians and within schools we see children unquestioningly placed into sets or streams according to some measure of attainment. We didn't bat an eyelid at statements such as, 'I was never any good at maths', 'I could never understand maths' or 'I just don't have a brain for maths'. I have also heard trainee teachers saying, 'I hated maths at school; my teacher only focused on the ones that he thought would get the good grades' and 'My teacher told me I didn't have the aptitude for maths and I believed her.'

However, the tide is changing. In his book, *Bounce*, international table-tennis champion Matthew Syed challenges this perception with countless examples of how hard work and perseverance are key to success. He even argues against the notion of child prodigies, explaining how Wolfgang Amadeus Mozart had practised the piano for something like 3,500 hours before he turned six.

*We see their little bodies and cute faces and forget that, hidden within their skulls, their brains have been sculpted – and their knowledge deepened – by practice that few people accumulate until well into adulthood, if then. Had the six-year-old Mozart been compared with musicians who had clocked up 3,500 hours of practice, he would not have seemed exceptional at all.*

(Syed, 2011, p53)

Carol Dweck's numerous research projects over several decades on mindset have challenged the notion of fixed intelligence (Dweck, 2006) and have had a profound effect on learning and teaching worldwide. Dweck classifies people into two distinct groups. People with a *fixed mindset* believe that their intelligence or talent are fixed traits and they believe that it is talent – without effort – that creates success. Children with a fixed mindset feel that they are either 'smart' or 'dumb' and because there is no way to change this, when they fail (which they of course do, often) they tell themselves they are no good at that subject/topic/skill. On the other hand, people with a *growth mindset* believe that *their most basic abilities can be developed through dedication and hard work – brains and talent are just the starting point. This view creates a love of learning and a resilience that is essential for great accomplishment* (Dweck, 2006). Children with growth mindsets believe that they can learn more or be smarter if they work hard and persevere. As a result, they learn more, learn it faster and view challenges and failures as opportunities to improve learning.

Jo Boaler in *Mathematical Mindsets* also argues against genius, stating that teachers *must work hard to break the myth of effortless achievement, pointing out that all high achievers have worked hard and failed often, even those thought of as geniuses* (Boaler, 2016, p178). The Programme for International Student Assessment (PISA) is a three-yearly study that looks at 15-year-olds' mathematics, science and reading performance. A clear finding from the 13 million children worldwide in the 2012 cohort was that the highest-achieving students in the world are those with a growth mindset and that they outperform the other students by at least a year of mathematics understanding (Boaler, 2016). In the next section we explore why this might be the case.

## Mindsets and mistakes

Earlier in this chapter we introduced Moser et al.'s (2011) research that used ERPs to measure what is going on in the brain as people make mistakes. Although it is interesting that people's brains react with greater responses to making a mistake – even when they aren't aware they had made one – than when they were getting the correct solution, the really interesting finding of this research is what happened in the brains of people who had a growth mindset compared to those who had a fixed mindset. The people with a growth mindset had greater brain activity than those with a fixed mindset. Furthermore, people with a growth mindset were more likely to have their synapses firing repeatedly, showing an awareness that a mistake had been made!

## The role of the teacher

This section considers the role of the teacher in light of the previous discussion. It is self-evident that such a shift will necessitate teachers adopting *a constructive attitude to their children's mistakes* (Koshy, 2000, p173) and children recognising that analysis and discussion of mistakes or misconceptions can be helpful to their mathematical development.

### Classroom socio-mathematical norms

Spooner (2002) suggests that placing children in situations where they feel in control of identifying mathematical errors/misconceptions leads to greater openness on the part of the children to explore and discuss their own misconceptions. Working with specifically designed pre-National Test materials for Key Stage 2, he discusses children exploring answers produced by an 'unknown pupil'. In order to do so they had to engage with the mathematical question set, discuss the errors with peers and explore why the error/misconception may have been made. The children appeared willing to engage in such discussions as it wasn't 'their' work under investigation. The process allowed children to be placed in the role of the teacher, encouraged open dialogue and consolidated their understanding of the concepts which underpinned the given examples.

This approach has an underlying belief that children's mathematical understanding is more likely to be developed if they are given opportunities to:

- explain their thinking;
- compare their thinking with that of peers and teachers.

To be effective in terms of long-term gains these opportunities need to be embedded within a school and classroom 'culture' to create socio-mathematical norms which accept and promote children's belief that they can learn effectively from their peers and need encouragement to 'be brave' to express their mathematical ideas. Support for this can be found in a report by Ofsted into the Primary National Strategies and their impact upon the rest of the primary curriculum.

*The most effective teachers . . . cultivate an ethos where children do not mind making mistakes because errors are seen as part of the learning. In these cases, children are prepared to take risks with their answers.*

(Ofsted, 2003, p18)

A significant feature of such approaches would be a recognition by children that learning often involves having to 'shift' one's thinking.

Thinking about the process of learning as one which can be mediated in school by the teacher may be helpful. It is likely that you will have already come across Vygotsky's

(1978) zone of proximal development (ZPD). It is defined as *the distance between the actual development level as determined by independent problem-solving and the level of potential to development as determined through problem-solving through adult guidance or in collaboration with more capable peers* (Vygotsky, 1978, p86). The notion of the ZPD was developed by Wood et al. (1979), who considered how teachers and peers could build (scaffold) and withdraw (fade) support as necessary to help a child bridge the ZPD.

Misconceptions can become rigid and resistant to revision later on (Furani, 2003). Therefore, it is the role of teachers to be aware of potential misconceptions and the possible reasons why they have developed. Swan (2001, p150) believes that mistakes and misconceptions *should be welcomed, made explicit, discussed and modified* if long-term learning is to take place. He suggests that this is unlikely to happen unless the teacher and children negotiate the *social nature of the classroom* and establish a classroom ethos based on trust, mutual support and value of individual viewpoints: there is a recognition that this is not easy and could result in teacher loss of confidence through apparent reduction in ‘control’ and reduction in the amount of ‘work’ produced on paper as more emphasis is placed on discussion and noisier classrooms.

The issue of a child’s maturity level to be able to deal with conflicting points of view or engage in mathematical dialogue is an important one and may lead teachers in Foundation Stage settings and Key Stage 1 classrooms to believe such an approach ‘unworkable’. It is interesting to note, however, that effective learning in mathematics appears to be connected with a school policy on an expectation that all children within the primary school will explain their mathematical ideas and methods (Askew et al., 1997). The notion of provoking cognitive conflict is not alien to a young child’s mathematical learning experiences: consider the ‘conflict’ caused by noticing that a ‘small’ object appears to be heavier than a ‘big’ object, a ‘tall’ container holds less water than a ‘short’ container or that the digit 4 can be ‘worth’ different amounts depending on the position of that digit in, for example, two- or three-digit numbers. The following chapters provide further discussion on this.

The development of a learning culture within classrooms is fundamental for teachers and children alike to explore how challenge can be an opportunity for new learning to take place. Hughes and Vass (2001) suggest that teacher language needs to be supportive in this respect: they identify the types of teacher language which would be helpful in supporting and motivating children to take risks in their learning. For example:

- the language of success – I know you can do it;
- the language of hope – you can do it and what help do you need to do it?
- the language of possibility – supporting a climate of greater possibility by the choice of response comment – yes, you did get it a bit mixed up but let’s see which bit is causing you problems.

Ryan and Williams (2007, p27) take this further by suggesting that teachers and pupils should view themselves as belonging to a *community of enquiry* in which the notion of *persuasion with coherent reasoning* is the norm. In such a community pupils are encouraged to share responsibility for sustaining the dialogue.

A key factor appears to be the 'control' and use made of teacher/pupil and pupil/pupil mathematical dialogue. Effective teachers of numeracy (Askew et al., 1997) encourage both types of dialogue, allow it to be sustained and use the results to help establish and emphasise connections and address misconceptions.

## Using talk to address misconceptions

A seminal research project into effective teaching and teachers of numeracy (Askew et al., 1997) highlighted that one factor involved in effective teaching was the emphasis placed on child/teacher discussion. In a school deemed to be one of the most effective in the teaching of numeracy there was a consistent expectation through Key Stage 1 and Key Stage 2 that children would develop skills in explaining their thinking processes: lessons in this school were characterised by dialogue in which teacher and children had to listen carefully to what was being said by others. Significant to this approach was a teacher belief, described as a connectionist belief, which views mathematics teaching and learning as something based on a dialogue between teacher and children and is characterised by extensive use of focused discussions in practice. Such a belief has connections with social constructivist perspectives on social and cultural dimensions to learning in which it is recognised that children can learn effectively from others, including their peers. Since this project, many others have espoused the importance of talk for learning mathematics (Williams, 2008; Bagnall, 2011; Ofsted, 2011; Mercer and Hodgkinson, 2008).

One concern here for trainee teachers and experienced teachers alike could be the possibility of peer discussion and/or peer collaboration compounding existing mathematical errors and misconceptions through persuasive dialogue. However Anghileri (2000) refutes the notion that common errors or misconceptions will be 'spread' among children through discussion: rather, she suggests that such activities will encourage children to review their thinking, leading to self-correction. Pound's research in Early Years settings supports this and found that peer discussion in play situations provided opportunities for rehearsing misconceptions. Through such rehearsals – providing that there were no interjections from adults – children develop better understandings as they *are able to challenge their own and other children's misconceptions* (Pound, 2008, p70).

The value in listening to explanations and the reasoning of others is viewed not only in the benefits to the restructuring of the specific and immediate mathematical idea, but also in the overall contribution to the development of individual mathematical thinking. This would suggest that the skills involved in using logic, reasoning, communication and problem-solving – the very skills inherent in children's ability to

use and apply mathematics – are actively developed by teaching beliefs and approaches which are deemed as connectionist (Williams, 2008).

Tanner and Jones (2000) suggest that restructuring thinking to accommodate new knowledge is not easy. In Piagetian terms this presents the children with *uncomfortable learning*, as previously assimilated knowledge has to be revisited, reshaped and challenged. In order for this to happen Tanner and Jones suggest that:

- children need to accept and appreciate that their response is not quite right;
- the learning process and environment need to be of sufficient importance to the children in order for them to make the effort to restructure and change their thinking;
- teachers need to accept that just explaining the misconception is not enough – the children will need help in the restructuring process.

The above is referred to as teaching for *cognitive conflict*: this describes children presented with examples and problems which lead to illogical outcomes. An example could be the addition of fractions  $\frac{1}{2} + \frac{1}{4}$ . . . : if the strategy of ‘add across top and bottom’ is applied, this result ( $\frac{2}{6}$ ) can be compared to a demonstration of a bar of chocolate where  $\frac{1}{2}$  is given to pupil A and  $\frac{1}{4}$  is given to pupil B – how much is left? ( $\frac{1}{4}$ ). The two different answers to the same example create conflict between existing conceptual understanding (to add fractional values just ‘add across’) and new information which challenges this existing framework. This conflict can be resolved through peer discussion, sharing of ideas, justifying responses, listening to others and teacher questioning. *Accommodation* can only occur when restructuring takes place within one’s schema to deal with this cognitive conflict.

Ryan and Williams (2007, p13) argue that misconceptions are often *intelligent constructions that should be valued by learners and teachers alike* and, as such, suggest that what is needed is a *related teaching design or strategy that engages or conflicts with the underlying misconception and reasoning directly* (p16). However, Ofsted (2009, p5) continues to report that in primary mathematics *most lessons do not emphasise mathematical talk enough; as a result, pupils struggle to express and develop their thinking* and that teachers *did not show enough urgency in checking whether each pupil had started the work correctly, had shown any of the expected misconceptions or was being challenged enough* (p19). In contrast, they identify that *the best teachers focus on pupils’ errors as a learning point. They spot the significant misconceptions which are illuminated by pupils’ mistakes* (p41).

Ofsted (2003) noted that one characteristic of mathematics lessons they deemed unsatisfactory was a tendency for teachers to do most of the talking. This resulted in children having *too few opportunities to try out their ideas orally, testing their thinking against that of others*. Where teachers used oral work well, they were more likely to:

- *discover and deal with errors or misconceptions and adjust their teaching in the light of these;*
- *help children to reflect on and sort out ideas and confirm their own understanding.*

(Ofsted, 2003, p18)

Skilful questioning can have the additional benefit of providing opportunities for children to engage in creative thinking and responses in mathematics (Briggs and Davis, 2008). Listening to children's questions also provides opportunities to gain insights into levels of understanding, errors in use of terminology and underlying misconceptions. Providing children with a diet of closed questions or tasks is therefore unlikely to allow teachers to ascertain children's errors or misconceptions.

## Formative and diagnostic assessment

Ofsted (2009, p4) identifies that a good teacher of mathematics recognises *quickly when pupils already understand the work or what their misconception might be. They extend thinking through building on pupils' contributions, questions and misconceptions to aid learning, flexibly adapting to meet needs and confidently departing from plans.* Shalem et al. (2014) offer a framework we can use to unpick our ability to address misconceptions that includes six criteria that teachers should consider:

1. Procedural understanding of the correct answer. How good is the quality of your procedural explanations when you are discussing the solution?
2. Conceptual understanding of the correct answer. How good is the quality of the conceptual links you make when you are discussing the solution?
3. Awareness of error. How good is the quality of your explanations as you focus on the error itself (not on the child's reasoning)?
4. Diagnostic reasoning of learners' thinking in relation to error. How good is the quality of your ability to explain the child's reasoning as they made the error?
5. Use of everyday links in explanations of error. How good is the quality of the everyday links you use to explain the error.
6. Multiple explanations of error. How are you at providing several explanations of an error, so different children can access the error in the way most appropriate for them?

When you are using the errors in later chapters to help plan for lessons, you can check your own understanding of the error using these six categories. Shalem et al. (2014) explain that it is important for teachers to be able to access all six aspects and that repeated pupil error is due to the first – procedural understanding – dominating teacher/pupil discussions.

Leighton et al. (2011) undertook a research project looking at diagnostic assessment which incorporated the Learning Errors and Formative Feedback (LEAFF) model. They identified the impact of children's performance when assessment is transparent or opaque (see Table 1.1).

When formative feedback is more transparent, knowledge and skill complexity increases and the number of errors decreases over time. However, when formative feedback is more opaque, errors increase in line with knowledge and skill complexity.

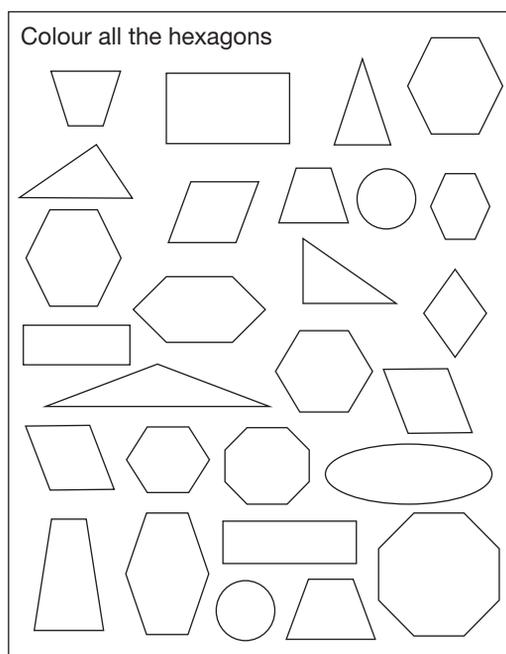
**Table 1.1** Impact of children’s performance

Formative feedback is transparent	Formative feedback is opaque
<ol style="list-style-type: none"> <li>1. More errors during formative assessments</li> <li>2. Greater pupil innovation and experimentation</li> <li>3. Higher-order thinking skills</li> <li>4. Higher interest and motivation</li> <li>5. Fewer errors on interim and summative assessments</li> </ol>	<ol style="list-style-type: none"> <li>1. Fewer errors during formative assessments</li> <li>2. Lesser pupil innovation and experimentation</li> <li>3. Lower-order thinking skills</li> <li>4. Lower interest and motivation</li> <li>5. More errors on interim and summative assessments</li> </ol>

Adapted from Leighton et al. (2011).

## Mathematical tasks

Cockburn (1999) discusses the nature of the mathematical tasks selected by the teacher as having potential for children to make errors: she suggests that consideration must be given to the complexity of the task (is it sufficiently challenging or too challenging?), the way the task is presented and the extent to which the child is able to *translate* the task, i.e. does the pupil know what is required in mathematical terms? This latter point is fundamental, for example, to a child’s ability to solve word problems. In addition to this, do the tasks you provide children ‘hide’ children’s misconceptions? Imagine providing your pupils with a sheet where they are required to colour all the hexagons such as the one in Figure 1.1. What might be the limitations of this task? How could you make it more appropriate for identifying, challenging and addressing children’s misconceptions in your classroom?



**Figure 1.1** Hexagons worksheet

Mason and Johnston-Wilder (2006, p64) explain that a *relevant type of task involves situations that appear to give rise to contradictions or surprises. In these tasks, learners need to sort out what is happening, resolve differences of opinion or conflicting explanations, and find some way to account for what is going on. Learners are called upon to explain things to each other and to locate differences and agreements in their explanations.* One of the criticisms of the worksheet in Figure 1.1 is that the only examples of hexagons are prototypical examples: they all have a horizontal baseline (as do the other examples on the sheet too). There is no opportunity for the children to explore other, non-prototypical shapes such as those in Figure 1.2. You could simply give the children a range of cards with different figures and ask them to group them into categories of their choosing. Look at the cards in Figure 1.2. There are a number of hexagons that are not prototypical, an open figure with six sides and other non-prototypical shapes. How do you think that this task would encourage children's discussion about hexagons, and likely elicit mathematical misconceptions, more than the worksheet?

Have a look at the Carroll Diagram in Figure 1.3. Consider each category according to tasks and your classroom culture. Where would you find the majority of the pupils in your classroom currently fit? Is this the group you would like them in, or is there a shift you need to make in your own teaching and approach to learning?

The next section brings together notions of talk and task by considering the mastery approach to mathematics.

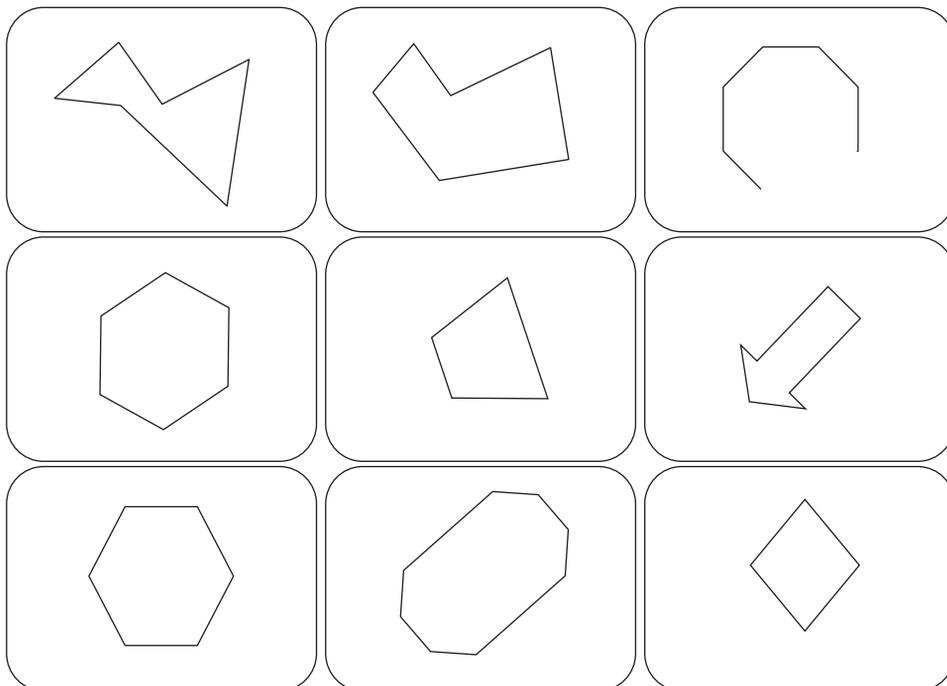


Figure 1.2 Selection of sort-it cards

	Misconception	No misconception
Error	Mistakes are caused by an underlying misconception. Misconceptions are systematically applied and a pattern can be seen in the mistakes made. The mistakes are not picked up by the child on re-checking.	Mistakes are caused by a 'silly slip' or an accidental miscalculation. They can be rectified if the child re-checks their work.
No error	No mistakes are made although the child does have an underlying misconception. The task presented is not providing an opportunity for the misconception to come to the fore.	There are no conceptual barriers the child has to the task they are carrying out. Therefore the work they have been given is not challenging enough and they are not learning anything new.

Figure 1.3 Misconceptions and errors Carroll Diagram

## Mathematics mastery

Mathematics mastery is a term used to describe approaches to teaching and learning mathematics that have seen east and southeast Asian countries such as China, Singapore, Japan and North Korea consistently top the international league tables in mathematics (NCETM, 2014). Although only recently introduced into schools, mastery appears to have the potential to improve pupil achievement in the UK also (Vignoles et al., 2015).

### Mindsets in mastery

Although specific mastery approaches differ between countries, there are underpinning principles common to all. These principles include the belief that all children are capable of learning mathematics. This means that teachers should not think of mathematical ability as being fixed, but rather as something that can be learned through a positive attitude and working hard. Topics are covered in more depth with a greater emphasis placed on problem-solving and mathematical thinking (Vignoles et al., 2015).

The aims of the national curriculum (DfE, 2013, p3) encourage the mastery approach to learning mathematics. Pupils who *grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.* Children should progress through curriculum content together, developing deep conceptual understanding alongside procedural fluency.

### Misconceptions in mastery

Given the way that topics are taught in detail and explored deeply, mastery provides space for an approach that actively encourages the following:

- explicit reference to and use of mistakes, misconceptions and misunderstandings;
- deep exploration of mathematical aspects that are not immediately clear;
- presentation of examples of concepts alongside 'non-examples' (for example distinguishing between shapes that are and aren't polygons) to provide children with the opportunity to contrast and compare.

Adapted from NCETM (2015)

Mastery involves taking time to develop deep knowledge of the key ideas that are needed to underpin future learning. This emphasises the structure and connections within mathematics so that children's learning can be sustained (NCETM, 2016). For example, taking time to learn – using various representations and contexts – about the effect multiplying and dividing by powers of ten has on numbers (and in particular on the value of digits in given places) challenges the misconception 'when multiplying by 10 you add a zero'.

## Summary

There needs to be a greater recognition from teachers that mathematical misconceptions are much more deeply rooted than errors and that it takes time for children and other learners to resolve long-held misconceptions. Teachers need to actively plan lessons which will confront children with carefully chosen examples that allow for challenge, dialogue and restructuring of thinking so they are able to understand.

Misconceptions are a natural part of a child's conceptual development and, consequently, greater time in mathematical lessons should be given to encouraging children to make connections between aspects of mathematical learning and their own meanings. The time needed for children's reflection, examination of their own ideas and comparison with those of other children and the mathematical situation presented, challenges the amount of mathematical content often covered in primary schools.

Regardless of the time allocated to mathematical discussion or activity, the culture of the classroom has to be one in which children are 'rewarded' for having the courage to test out their mathematical ideas in order for errors and misconceptions to be aired, discussed and resolved. If getting the right answer is the aim of the activity, or presenting the work in a neat way, or completing a set of exercises in a given time, then probing children's misunderstandings and misconceptions may prove difficult and counter-productive to effective mathematical learning.